

Solutions to Selected Exercises, HW #2

Assignment:

- S-POP, Part B(ii): Exercises B(ii)-1, 2, 3, 4.
- T-BOP, Section 1.2: Exercises A1(abgh), A2(bf), A3, A8.
- T-BOP, Section 1.3: Exercises A1, A6, C13, C15.
- T-BOP, Section 1.4: Exercises A2, A4, A5, B14, B15.
- T-BOP, Section 1.5: Exercises 1, 4ade.

S-POP, Part B(ii)

Exercise B(ii)-1. Show that the set of all integer multiples of 4 is contained in the set of all even numbers.

Proof. Let $A = \{\text{all integer multiples of } 4\} = 4\mathbb{Z}$, and let $B = \{\text{all even numbers}\} = 2\mathbb{Z}$. We wish to show that $A \subseteq B$.

So let $x \in A$. Then, by definition of A , $x = 4k$ for some $k \in \mathbb{Z}$. But $4 = 2 \cdot 2$, so

$$x = (2 \cdot 2)k = 2 \cdot (2k) = 2m,$$

where $m \in \mathbb{Z}$. So, by definition of B , $x \in B$.

So $A \subseteq B$. \square

Exercise B(ii)-3. Show that, for any sets S and T , we have $S \subseteq S \cup T$.

SOLUTION:

Proof. Let S and T be sets. Let $x \in S$. Then certainly $x \in S$ or $x \in T$, so $x \in S \cup T$, by definition of union.

So $S \subseteq S \cup T$. \square

T-BOP, Section 1.2

Exercise A2: (b) $B \times A = \{(0, \pi), (0, e), (0, 0), (1, \pi), (1, e), (1, 0)\}$.

(f) Technically, $(A \times B) \times B$ is the set of all ordered pairs (x, y) , where $x \in A \times B$ and $y \in B$. So technically, $(A \times B) \times B$ looks like this:

$$(A \times B) \times B = \{((\pi, 0), 0), ((\pi, 1), 0), ((e, 0), 0), ((e, 1), 0), ((0, 0), 0), ((0, 1), 0), \\ ((\pi, 0), 1), ((\pi, 1), 1), ((e, 0), 1), ((e, 1), 1), ((0, 0), 1), ((0, 1), 1)\}.$$

But it's OK to think of $(A \times B) \times B$ as being the same as $A \times B \times B$. So we could write

$$(A \times B) \times B = \{(\pi, 0, 0), (\pi, 1, 0), (e, 0, 0), (e, 1, 0), (0, 0, 0), (0, 1, 0), \\ (\pi, 0, 1), (\pi, 1, 1), (e, 0, 1), (e, 1, 1), (0, 0, 1), (0, 1, 1)\}.$$

Do you see the difference? It's subtle, and I'll accept either answer, but there is, technically, a distinction. See the bottom of p. 10 of T-BOP.

Exercise A8.

$$\{0, 1\}^4 = \{(0, 0, 0, 0), (0, 0, 0, 1), (0, 0, 1, 0), (0, 0, 1, 1), (0, 1, 0, 0), (0, 1, 0, 1), (0, 1, 1, 0), (0, 1, 1, 1), \\ (1, 0, 0, 0), (1, 0, 0, 1), (1, 0, 1, 0), (1, 0, 1, 1), (1, 1, 0, 0), (1, 1, 0, 1), (1, 1, 1, 0), (1, 1, 1, 1)\}.$$

T-BOP, Section 1.3

Exercise A6. $\emptyset, \{\mathbb{R}\}, \{\mathbb{Q}\}, \{\mathbb{N}\}, \{\mathbb{R}, \mathbb{Q}\}, \{\mathbb{Q}, \mathbb{N}\}, \{\mathbb{R}, \mathbb{N}\}, \{\mathbb{R}, \mathbb{Q}, \mathbb{N}\}.$

T-BOP, Section 1.4

Exercise A4. $\mathcal{P}(\{\mathbb{R}, \mathbb{Q}\}) = \{\emptyset, \{\mathbb{R}\}, \{\mathbb{Q}\}, \{\mathbb{R}, \mathbb{Q}\}\}.$

Exercise A14. $|\mathcal{P}(\mathcal{P}(A))| = 2^{|\mathcal{P}(A)|} = 2^{2^m}.$

Remark: 2^{2^m} means $2^{(2^m)}$, not $(2^2)^m$. They're not the same; for example, $2^{(2^3)} = 2^8 = 256$, while $(2^2)^3 = 4^3 = 64$.

T-BOP, Section 1.5

Exercise 4: (a) $A \times B = \{(b, a), (b, b), (c, a), (c, b), (d, a), (d, b)\}$ and $B \times B = \{(a, a), (a, b), (b, a), (b, b)\}$, so $(A \times B) \cup (B \times B) = \{(b, a), (b, b)\}.$

(d) $(A \cap B) \times A = \{1\} \times \{0, 1\} = \{(1, 0), (1, 1)\}.$

(e) $(A \times B) \cap B = \emptyset$, since $A \times B$ is a set of ordered pairs but B contains no ordered pairs.