Here we study relationships among numbers of vertices, edges, and faces in a polyhedron.

We begin with some definitions. First: a *simple closed surface* is an object, in three dimensions, that's hollow, has no holes, and whose interior is *connected*. (That is: you can get from any point inside the surface to any other point inside the surface without crossing any part of the surface itself.) A *polyhedron* is a simple closed surface made up of polygonal regions joined together. (*Polyhedra* is the plural of polyhedron.)

The polygonal regions making up a polyhedron are called *faces* of the polyhedron. The terms *vertices* and *edges*, when applied to a polyhedron, refer simply to the vertices and edges of the polygonal regions making up that polyhedron.

1. First, you need to *build* the polyhedron from the "net" (two-dimensional template) you were given in class. When you're done, COUNT the vertices, edges, and faces of your polyhedron; then enter the name of your polyhedron, as well as the numbers you counted, in the indicated spaces of the first row below.

In the next two rows below, enter the requested information for the polyhedra that a couple of your classmates built. (Make sure they're different from the one you built.) Then enter the same information for several of the many-sided dice you have available. (If you don't know the names of the polyhedra represented by these dice, make something up: call them "Fred" and "Lucinda," for example.)

Name of Polyhedron	Number of vertices (V)	Number of edges (E)	Number of faces (F)
Octahedron	6	12	8
Truncated Square Pyramid	8	12	6
Step Pyramid	9	16	9
Pentagonal Pyramid	6	10	6
Cube (die)	8	12	6
Dodecahedron (die)	20	30	12

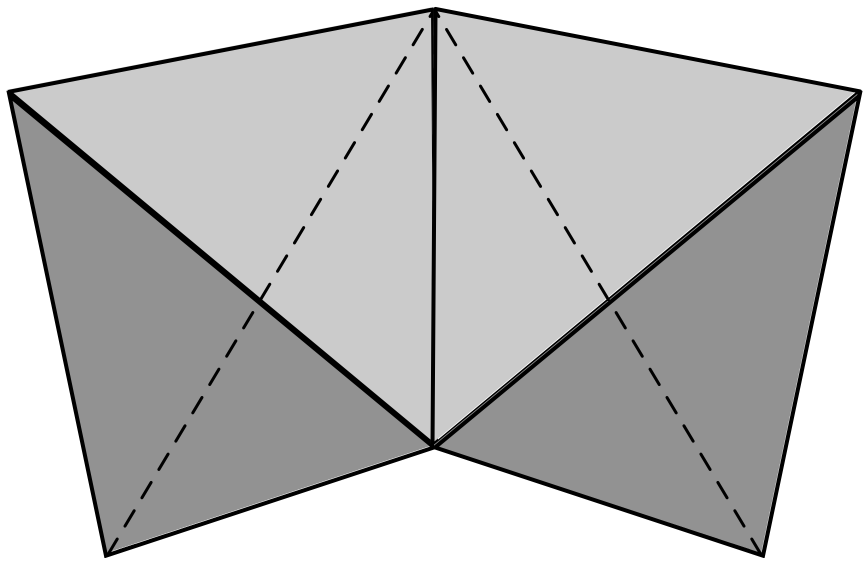
2. Using the information from the table you just completed, answer the following.

CONJECTURE ("Euler's Formula"): If P is any polyhedron, and V, E, and F represent the numbers of vertices, edges, and faces of P, respectively, then

$$\underline{\quad V - E + F \quad} = \underline{\quad 2 \quad}.$$

(The blank on the left hand side should be filled in with some simple expression involving V, E, and F, which you should be able to guess at by examining your table above. The blank on the right hand side should be filled in with a natural number.)

3. Use the sketch below of a "dual tetrahedron" (formed by gluing two tetrahedra together along an edge of each) to answer the given questions.



To count vertices, edges, and faces of the above object, note that each vertex and edge only counts once. For example, the edge along which the two tetrahedra are joined counts as one edge, not two.

How many vertices does the dual tetrahedron have? 6 How many edges? 11
How many faces? 8

Does this contradict your conjecture above? Explain. Hint: Note that the above conjecture starts with "If P is a polyhedron." Now carefully consider the definitions above (at the top of the first page).

No, it does not contradict the conjecture, because the dual tetrahedron is not a polyhedron. Why? Because the definition of a polyhedron given on the first page says that a polyhedron *does not cross itself anywhere,* meaning *you can get from any point inside the surface to any other point inside the surface without crossing any part of the surface itself.* But this is not true for the dual tetrahedron, because to get from a point inside one of the tetrahedra to a point inside the other, you need to cross the surface of the object.

In terms of logic and proofs and such: To deduce Q from a statement of the form "If P, then Q," you need to know that P holds. If P is not true, then Q may or may not be true.