Math 2001-004: Intro to Discrete **Name:**

Quiz, Week 9 SOLUTIONS

For this quiz, *do not* use a calculator. You can express your answers to exercises 1 and 2 in terms of sums/differences/products of natural numbers; you *do not* need to add/subtract/multiply things out. For exercises 3 and 4, *do* express each answer as a single natural number.

Please show your work and/or provide an explanation for each answer.

1. This problem concerns lists made from the letters A, B, C, D, E, F, G, H, I.

(a) How many length-6 lists can be made from these letters if repetition is not allowed and the list must contain (exactly) one D?

The number of ways of doing this if we assume that D is the first letter is: $1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$ (one choice for the first letter – since it must be D – leaving 8 choices for the next, 7 for the next, etc.). If we assume that D is the second letter, the number of ways is: $8 \cdot 1 \cdot 7 \cdot 6 \cdot 5 \cdot 4$ (eight choices for the first letter, 1 for the second, 7 for the next, etc.). We get the same product, but in different order, for each of the 6 places that can hold a D. So the total is

$$6 \cdot (1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4).$$

(b) How many length-5 lists can be made from these letters if repetition *is* allowed – that is, the same letter can appear more than once – but no two *consecutive* letters can be the same?

There are nine choices for the first letter. The second letter must be different from the first, so there are eight choices for the second. The third letter must be different from the second (but could be the same as the first), so there are eight choices for the third. And so on, yielding a total of

$$9 \cdot 8 \cdot 8 \cdot 8 \cdot 8 = 9 \cdot 8^4$$

possible lists.

2. Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such line-ups (lists) of cards are there in which all cards have *different* face values? (Recall that a standard deck has thirteen face values – Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K – and four cards of each face value.)

$$52 \cdot 48 \cdot 44 \cdot 40 \cdot 36$$

(there are 52 choices for the first card, leaving 48 for the next, leaving 44 for the next, and so on).

3. Find the value of $\frac{20!}{18!}$, and express this answer as a single natural number.

$$\frac{20!}{18!} = 20 \cdot 19 = 380.$$

4. Express $\frac{12 \cdot 10 \cdot 8 \cdot 6 \cdot 4 \cdot 2}{6! \cdot 2^6}$ as a single natural number.

$$\begin{aligned}\frac{12 \cdot 10 \cdot 8 \cdot 6 \cdot 4 \cdot 2}{6! \cdot 2^6} &= \frac{(2 \cdot 6) \cdot (2 \cdot 5) \cdot (2 \cdot 4) \cdot (2 \cdot 3) \cdot (2 \cdot 2) \cdot (2 \cdot 1)}{6! \cdot 2^6} \\ &= \frac{(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) \cdot 2^6}{6! \cdot 2^6} = \frac{6! \cdot 2^6}{6! \cdot 2^6} = 1.\end{aligned}$$