Math 2001-004: Intro to Discrete **Name: SOLUTIONS**

Quiz, Week 6

1. Use the Principle of Mathematical Induction (MI) to prove the following.

Proposition For all $n \in \mathbb{N}$, we have

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$

Proof. Let A_n be the statement in question.

Step 1: Base step. [*Please complete the base step in the space below.*]

$$1^3 = \frac{1^2(1+1)^2}{4}?$$
$$1 = 1.$$

So A_1 is true.

Step 2: Inductive step. We assume

$$A_k\colon 1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{k^2(k+1)^2}{4}.$$

To deduce

$$A_{k+1}\colon 1^3 + 2^3 + 3^3 + \cdots + (k+1)^3 = \underline{\frac{(k+1)^2(k+2)^2}{4}}$$

[*please write the right hand side of A_{k+1} in the above blank space*], we note the following. [*Please complete the inductive step in the space following the second equal sign below. It may help, at some point, to get a common denominator, and* then *to factor $(k+1)^2$ out of the numerator. There's more space on the back of this sheet, if you need it.*]

$$1^3 + 2^3 + 3^3 + \cdots + (k+1)^3 = [1^3 + 2^3 + 3^3 + k^3] \cdots + (k+1)^3$$
$$= \frac{k^2(k+1)^2}{4} + (k+1)^3 = \frac{k^2(k+1)^2}{4} + \frac{4(k+1)^3}{4} = \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$$
$$= \frac{(k+1)^2(k^2 + 4(k+1))}{4} = \frac{(k+1)^2(k^2 + 4k + 4)}{4} = \frac{(k+1)^2(k+2)^2}{4}.$$

So $A_k \Rightarrow$ $\underline{\quad A_{k+1} \quad}$ [*please fill in the blank*].

So, by the principle of __mathematical induction__, A_n is true for all $n \in$ __$\mathbb{N}$__ [*please fill in the blanks*], and our proposition is proved. **ATWMR**

2. Suppose we have some statement A_n regarding integers $n \in \mathbb{N}^-$, where $\mathbb{N}^-$ denotes the set of *negative* integers n. Suppose we know that A_{-1} is true. What else would we have to show, to prove that our statement holds for *all* $n \in \mathbb{N}^-$? Please explain. (You don't need to prove anything here; just explain the basic, intuitive ideas.)

If we know A_{-1} is true, then to prove A_n for all $n \in \mathbb{N}^-$, we must show that $A_n \Rightarrow A_{n-1}$ for all negative integers n. In this way, A_{-1} will imply A_{-2}, which will imply A_{-3}, and so on down the line.