Math 2001-004: Intro to Discrete **Name: SOLUTIONS**

Quiz, Week 5

1. According to some definitions of "divides" (like the one in our text; see page 90), zero *does* divide zero; according to other definitions that you see sometimes, it doesn't. Which of the following definitions is which? For each definition, state whether zero does or doesn't divide zero, and explain thoroughly.

Definition A. (This is the definition we will always use.) For $m, n \in \mathbb{Z}$, we say m *divides* n, written $m|n$, if $\exists$ some integer k such that $n = mk$.

Yes, zero does divide zero, because there does exist an integer k such that $0 = k \cdot 0$. In fact, any integer k will work.

Definition B. For $m, n \in \mathbb{Z}$, we say m *divides* n, written $m|n$, if $\exists$ a *unique* integer k (that is, there is only *one* integer k) such that $n = mk$.

No, zero does not divide zero, because there exists more than one integer k such that $0 = k \cdot 0$. Again, any integer k will work.

2. Identify each of the following statements as true or false (circle "**T**" or "**F**"). If a statement is true, explain why (you don't need to provide a complete proof; just a sentence or two will do). If a statement is false, provide a counterexample to the statement.

(a) $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z} \colon m|n$. **T** **F**

Given $n \in \mathbb{Z}$, let $m = 1$ (or $m = n$). Then $m|n$.

(b) $\exists n \in \mathbb{Z} \colon \forall m \in \mathbb{Z}, m|n$. **T** **F**

Let $n = 0$. Then for any $m \in \mathbb{Z}$, we have $m|n$.

(c) $\forall m \in \mathbb{Z}, \forall n \in \mathbb{Z}, \exists k \in \mathbb{Z} : (m - n)|k$. **T** **F**

Given $m, n \in \mathbb{Z}$, let $k = 0$ or $k = m - n$. Then $(m - n)|k$.

(over)

2. Fill in the blanks, to prove the following:

Proposition. Given any sets A, B, and C, we have

$$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C).$$

Proof. Let $x \in (A \cup B) \cap (A \cup C)$. We are going to deduce that $x \in A \cup (B \cap C)$ by considering two cases: $x \in A$ and $x \notin A$. Certainly, one of these cases must hold, so proving that the desired result holds in each of these two cases will prove that it holds in general.

(i) $x \in A$. But then it's certainly true that $x \in A$ *or* $x \in B \cap C$. So by definition of ___union___, we have $x \in A \cup$ ___$B \cap C$___, as we wanted to show.

(ii) $x \notin A$. Note that, by our assumption that $x \in (A \cup B) \cap (A \cup C)$, we have $x \in A \cup B$ and $x \in$ ___$A \cup C$___. The first of these facts (that is, the fact that $x \in A \cup B$), together with the assumption $x \notin A$, tells us that $x \in B$; the second of these facts (that is, the fact that $x \in A \cup C$), together with the assumption $x \notin A$, tells us that $x \in$ ___C___. Combining these last two facts (that is, the facts that $x \in B$ and $x \in C$), we conclude that $x \in B \cap$ ___C___. But then, by definition of ___union___, we have $x \in A \cup (B \cap C)$, as we wanted to show.

In either case, we have $x \in A \cup (B \cap C)$. So ___$(A \cup B) \cap (A \cup C)$___ $\subseteq A \cup (B \cap C)$.

___ATWMR___

(Supply your own tag line in the last blank.)