Math 2001-004: Intro to Discrete **Name: SOLUTIONS**

Quiz, Week 4

1. For this problem, you may want to recall that, for $m, n \in \mathbb{Z}$, we say m *divides* n, written $m|n$, if there is some integer k such that $n = mk$.
Identify each of the following statements as true or false (circle "**T**" or "**F**"). If a statement is true, explain why (you don't need to provide a complete proof; just a sentence or two will do). If a statement is false, provide a counterexample to the statement. GENERAL HINT: think about zero and one.

(a) $\forall m \in \mathbb{Z}, \exists n \in \mathbb{Z} \colon m|n.$ **T** **F**

Proof: Let $m \in \mathbb{Z}$. Then $m|0$, since $0 = m \cdot 0$. So $\exists n \in \mathbb{Z} \colon m|n$.

(b) $\exists m \in \mathbb{Z} \colon \forall n \in \mathbb{Z}, m|n.$ **T** **F**

Proof: Let $m = 1$, and let $n \in \mathbb{Z}$. Then $m|n$, since $n = 1 \cdot n$. So $\forall n \in \mathbb{Z} \colon m|n$.

(c) $\forall m \in \mathbb{Z}, \forall n \in \mathbb{Z}, m|n.$ **T** **F**

Proof: Let $m = 2$ and $n = 3$. Then $m \nmid n$.

(over)

2. Fill in the blanks, to prove the following:

Proposition. Given any sets A, B, and C, we have

$$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C).$$

Proof.
Let $x \in A \cup (B \cap C)$. Then, by definition of union, either $x \in A$ or $x \in$ ___$B \cap C$___. We consider two cases:

(i) $x \in A$. But then, by definition of union, we have $x \in A \cup B$ **and** $x \in A \cup C$. So by definition of ___intersection___, we have $x \in (A \cup B) \cap$ ___$(A \cup C)$___.

(ii) $x \in B \cap C$. But then, by definition of intersection, we have $x \in B$ **and** $x \in$ ___C___. So by definition of ___union___, we have $x \in A \cup B$, and, similarly, $x \in$ ___$A \cup C$___. So by definition of ___intersection___, we have $x \in$ ___$(A \cup B)$___ $\cap (A \cup C)$.

In either case, we have $x \in (A \cup B) \cap (A \cup C)$. So ___$A \cup (B \cap C)$___ $\subseteq (A \cup B) \cap (A \cup C)$.

ATWMR

3. To prove that, for any sets A, B, and C, we have

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C),$$

what else (besides the above proposition) would you need to prove? Please just provide the *statement* that you would need to complete the proof; you *don't* need to supply the proof of that statement (well, not until your midterm, anyway).
We would need to show that

$$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C).$$