Math 2001-004: Intro to Discrete **Name:** SOLUTIONS

Quiz #2

1. Let
$$S = \{\sqrt{2}, e, \pi\}, \quad T = \{-273, e, \pi, 7\}, \quad U = \{-273, \sqrt{2}, e, 29\}.$$

(a) Find $(S - U) \cap T$.

$$\begin{aligned}(S-U)\cap T &= (\{\sqrt{2}, e, \pi\} - \{-273, \sqrt{2}, e, 29\}) \cap \{-273, e, \pi, 7\} \\ &= \{\pi\} \cap \{-273, e, \pi, 7\} = \{\pi\}.\end{aligned}$$

(b) Find $S - (U \cap T)$.

$$\begin{aligned}S-(U\cap T) &= \{\sqrt{2}, e, \pi\} - (\{-273, \sqrt{2}, e, 29\} \cap \{-273, e, \pi, 7\}) \\ &= \{\sqrt{2}, e, \pi\} - \{-273, e\} = \{\sqrt{2}, \pi\}.\end{aligned}$$

(c) What is $|\mathscr{P}(T \times U)|$? Note that you're not actually being asked to *describe*, or specify the elements of, $\mathscr{P}(T \times U)$. The question is simply *how many* elements this set has. Please explain your answer.

We know that $T \times U$ has $|T| \times |U| = 4 \times 4 = 16$ elements. Now the power set of a set with m elements has 2^m elements, so $|\mathscr{P}(T \times U)| = 2^{16}$.

(over)

2. Prove the following:

Proposition. $7 + 15\mathbb{Z} \subseteq 7 + 5\mathbb{Z}$.

Proof. Let $x \in 7 + 15\mathbb{Z}$. Then $x = 7 + 15k$ for some integer k. Since $15 = 5 \cdot 3$, we have $x = 7 + (5 \cdot 3)k = 7 + 5(3k) = 7 + 5c$, where $c = 3k \in \mathbb{Z}$. So $x \in 7 + 5\mathbb{Z}$.

So $7 + 15\mathbb{Z} \subseteq 7 + 5\mathbb{Z}$. ATWMR