Math 2001-004: Intro to Discrete **Name: SOLUTIONS**

Quiz #1

Recall that, for $a, b \in \mathbb{Z}$, we denote by $a + b\mathbb{Z}$ the set

$$a + b\mathbb{Z} = \{n \in \mathbb{Z} : n = a + bk \text{ for some } k \in \mathbb{Z}\}.$$

1. Describe $3 + 5\mathbb{Z}$, by listing some elements and using ellipses (that is, $\dots$).

$$3 + 5\mathbb{Z} = \{\dots, -12, -7, -2, 3, 8, 13, \dots\}$$

2. Describe

$$S = \{n \in -5 + 11\mathbb{Z} \colon -40 < n < 55\}$$

in the form of a list of elements (inside braces). What is $|S|$?

$$S = \{-38, -27, -16, -5, 6, 17, 28, 39, 50\}.$$

$$|S| = 9.$$

(over)

3. Prove the following:

Proposition. If $n \in 6 + 14\mathbb{Z}$, then $n \in 3 + 7\mathbb{Z}$.

DOH!!! What I asked you to prove is FALSE!!! Here's a counterexample: Let $n = 6$. Then certainly $n \in 6+14\mathbb{Z}$, because $n = 6+14\cdot 0$. But $n \notin 3+7\mathbb{Z}$, because the equation $6 = 3+7k$ would give $3 = 7k$, which is impossible because $7 \not| 3$.

Here's what I *meant* to ask, and how to prove it.

Proposition. If $n \in 6 + 14\mathbb{Z}$, then $n \in 6 + 7\mathbb{Z}$.

Proof. Suppose $n \in 6 + 14\mathbb{Z}$. Then $n = 6 + 14k$ for some integer k. But then $n = 6 + (7 \cdot 2)k = 6 + 7 \cdot (2k) = 6 + 7c$, where $c = 2k \in \mathbb{Z}$. So $n \in 6 + 7\mathbb{Z}$.

So $n \in 6 + 14\mathbb{Z} \Rightarrow n \in 6 + 7\mathbb{Z}$. ATWMR

4. Is the *converse* of the above Proposition true? That is: is it true that $n \in 3 + 7\mathbb{Z} \Rightarrow n \in 6 + 14\mathbb{Z}$? If this is true, prove it. If not, provide a counterexample.

The converse to neither the original (false) proposition nor the corrected (true) proposition is true.

Here's a counterexample to the statement $n \in 3 + 7\mathbb{Z} \Rightarrow n \in 6 + 14\mathbb{Z}$: Let $n = 3$. Then $n = 3 + 7 \cdot 0$, so $n \in 3 + 7\mathbb{Z}$. But $n \notin 6 + 14\mathbb{Z}$, because the equation $3 = 6 + 14k$ would give $-3 = 14k$, which is impossible because $14 \not| (-3)$.

Here's a counterexample to the statement $n \in 6 + 7\mathbb{Z} \Rightarrow n \in 6 + 14\mathbb{Z}$: Let $n = 13$. Then $n = 6 + 7 \cdot 1$, so $n \in 6 + 7\mathbb{Z}$. But $n \notin 6 + 14\mathbb{Z}$, because the equation $13 = 6 + 14k$ would give $7 = 14k$, which is impossible because $14 \not| 7$.