Math 2001-004: Intro to Discrete **Name:** SOLUTIONS

Quiz, Week 14

1. Complete the proof below.

Proposition. Define $f\colon \mathbb{R}-\{3\}\to\mathbb{R}-\{1\}$ by

$$f(x)=\frac{x+2}{x-3}.$$

Then f is a bijection.

Proof. First we show that f is injective: let $x_1,x_2\in\mathbb{R}$ and suppose $f(x_1)=f(x_2)$. Then

$$\frac{x_1+2}{x_1-3}=\frac{x_2+2}{x_2-3},$$

so [fill in the rest:]

$$\begin{aligned}(x_1+2)(x_2-3)&=(x_2+2)(x_1-3),\\ x_1x_2-3x_1+2x_3-6&=x_1x_2-3x_2+2x_1-6,\\ -3x_1+2x_3&=-3x_2+2x_1,\\ -5x_1&=-5x_2,\\ x_1&=x_2.\end{aligned}$$

So $f(x_1)=f(x_2)\Rightarrow x_1=x_2$. So f is injective.

Next, we show that f is surjective: let $y\in\mathbb{R}-\{1\}$. We need to find $x\in\mathbb{R}-\{3\}$ such that [fill in the rest:] $f(x)=y$. That is, we need

$$\begin{aligned}y&=\frac{x+2}{x-3},\\ y(x-3)&=x+2,\\ xy-3y&=x+2,\\ xy-x&=2+3y\\ x(y-1)&=2+3y,\\ x&=\frac{2+3y}{y-1}.\end{aligned}$$

So $\exists x\in\mathbb{R}-\{3\}$ with $f(x)=y$. So f is surjective. So f is bijective.

ATWMR

2. Complete the proof below, by filling in the blanks.

Proposition. If S is any set and $A \subseteq S$, then $|A| \leq |S|$.

Proof. Let S be any set, and assume $A \subseteq$ __S__. By definition of the statement $|A| \leq |S|$, we need only find an injection $f\colon A \to$ __S__. But this is easy: define $f\colon$ __A__ $\to S$ by $f(a) = a$. To show that f is injective, let $a_1, a_2 \in A$, and suppose $f(a_1) = f($__a_2__$)$. By definition of f we have $f(a_1) = a_1$ and $f(a_2) =$ __a_2__, so we conclude that __a_1__ $= a_2$.

So $f(a_1) = f(a_2) \Rightarrow$ __$a_1 = a_2$__. So f is __injective__, and our proposition is proved. **ATWMR**