Math 2001-004: Intro to Discrete **Name: SOLUTIONS**

Quiz, Week 10

For this quiz, *do not* use a calculator. You can express your answers in terms of sums/ differences/products of natural numbers; you *do not* need to add/subtract/multiply things out. (Though you can if you want.)

Please show your work and/or provide an explanation for each answer.

1. A set X has 66 subsets S with $|S| = 2$.

(a) What is $|X|$? **Please show your work and/or provide an explanation for your answer.**

Write $|X| = n$. We have

$$\binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2} = 66.$$

This gives $n(n-1) = 2 \cdot 66 = 132$, or $n^2 - n - 132 = 0$, or $(n+11)(n-12) = 0$, or $n = -11$ or $n = 12$. Since n must be positive, we have $n = 12$.

(b) How many subsets T of X contain exactly 4 elements? **Please show your work and/or provide an explanation for your answer.** Please express your answer either as a single natural number, or as a product of natural numbers (that is, cancel out any denominators).

$$\binom{12}{4} = \frac{12!}{4!8!} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = 11 \cdot 5 \cdot 9 = 495.$$

There are 495 subsets of X with 4 elements.

2. This problem concerns lists made from the seven digits 1, 2, 3, 4, 5, 6, 7, without repetition.

(a) How many such lists of length *seven*, from these digits, have the property that the 3 occurs before the 6? **Please show your work and/or provide an explanation for your answer.**

We choose two slots, in our list of length seven, to place the 3 and the 6; there are $\binom{7}{2} = 21$ ways to do this. There are 5! ways of filling the remaining five slots. This gives a total of $21 \cdot 5!$ lists with the given property.

(b) How many such lists of length *six*, from these digits, contain either a 3 or a 6 (or both)? **Please show your work and/or provide an explanation for your answer.**

All $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$ lists of length six must have either a 3 or a 6 or both, because otherwise we're leaving out two of the seven digits, leaving us with only five digits to choose from, and how could we have a list of length six (without repetition) made out of only five digits?