Week 4 - Friday, 9/18

CSM.

Differentiation <u>formulas</u> and <u>rules</u>

(I) Formulas.

Using the definition

$$(*)\qquad f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x},$$

we can derive the following formulas:

	$f(x)$	$f'(x)$
(A_0)	1	0
(A_1)	x	1
(A_2)	x^2	$2x$
(A_3)	x^3	$3x^2$
(A)	x^p (p a constant)	px^{p-1}
(B)	$\sin(x)$	$\cos(x)$
(C)	$\cos(x)$	$-\sin(x)$
(D)	$\tan(x)$	$\sec^2(x)$ ← definitions:

$\tan(x) = \dfrac{\sin(x)}{\cos(x)}$

$\sec(x) = 1/\cos(x)$

$\sec^2(x)$ means $(\sec(x))^2$

We'll also need a formula for <u>exponential functions</u>, meaning functions like

$f(x) = b^x$ (b a positive constant)

Examples: $f(x) = 2^x$, $f(x) = (1/3)^x$, $f(x) = \pi^x$, etc.

Let's do it: if $f(x) = b^x$, then

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{b^{x+\Delta x} - b^x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{b^x b^{\Delta x} - b^x}{\Delta x} = \lim_{\Delta x \to 0} \frac{b^x(b^{\Delta x} - 1)}{\Delta x}$$

$$= b^x \lim_{\Delta x \to 0} \frac{b^{\Delta x} - 1}{\Delta x}$$

Call this number k_b: it depends only on b!!

b^x does not depend on Δx, so pull b^x out front

To summarize:

(E) If $f(x) = b^x$, then

$f'(x) = k_b \cdot b^x$ for some constant b.

Notes:

(1) In other words: the rate of change of an exponential function is proportional to the magnitude of the function itself.

(2) For any given b, k_b can be approximated by

$$k_b \approx \frac{b^{\Delta x} - 1}{\Delta x},$$

for Δx small.

Examples:

we compute that

$$k_2 \approx \frac{2^{0.00001}-1}{0.0001} = 0.6931\ldots,$$

$$k_3 \approx \frac{3^{0.00001}-1}{0.00001} = 1.0986\ldots,$$

$$k_{1/4} \approx \frac{\left(1/4\right)^{0.00001}-1}{0.00001} = -1.3862\ldots,$$

etc.

II) Rules.

Let's write

$$\frac{d}{dx}[f(x)] \text{ for } f'(x).$$

E.g. $\frac{d}{dx}[x^3] = 3x^2$; $\frac{d}{dx}[3^x] = k_3 \cdot 3^x$;

etc.

Then we have:

(A) The "constant multiple" rule:

$$\frac{d}{dx}[cf(x)] = cf'(x) \text{ for any constant } c.$$

(B) The "sum" rule:

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x).$$

(III) Examples (DIY: what formulas/rules are we using?)

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(i) $\frac{d}{dx}[x^{25}] = 25x^{24}$.

(ii) $\frac{d}{dx}\left[\frac{1}{x^{25}}\right] = \frac{d}{dx}[x^{-25}] = -25x^{-26} = \frac{-25}{x^{26}}$.

(iii) $\frac{d}{dx}\left[\sqrt[25]{x}\right] = \frac{d}{dx}\left[x^{1/25}\right] = \frac{1}{25}x^{\frac{1}{25}-1}$

$= \frac{1}{25}x^{-24/25} = \frac{1}{25\cdot\sqrt[25]{x^{24}}}$.

(iv) $\frac{d}{dx}[17x^{25}] = 17\cdot\frac{d}{dx}[x^{25}] = 17\cdot 25x^{24}$

$= 425x^{24}$.

(v) $\frac{d}{dx}[x^{12} + 5\sin(x)] = \frac{d}{dx}[x^{12}] + 5\frac{d}{dx}[\sin(x)]$

$12x^{11} + 5\cos(x)$.

(vi) $\frac{d}{dx}[x^{\pi} + \pi^{x}]$

$= \pi x^{\pi-1} + k_{\pi}\cdot\pi^{x}$.

(vii) $\frac{d}{dx}[\sin(x^2)] = ?$

Come back Monday.