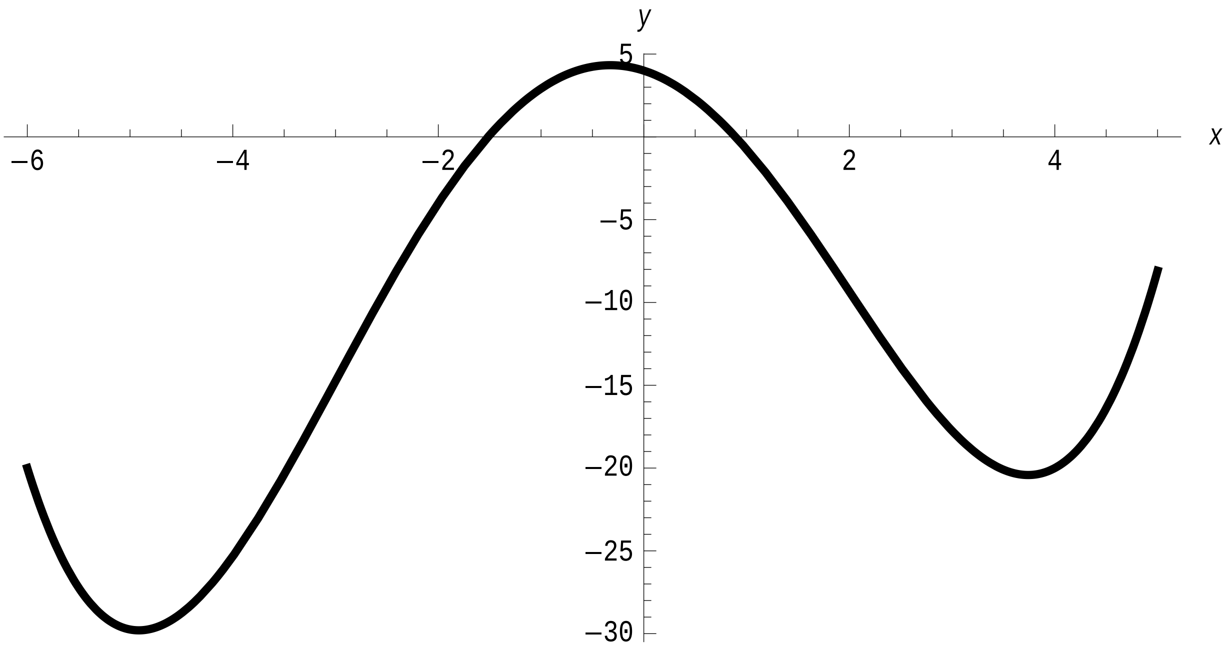
On the axes below is the graph of a certain function $f(x)$.



1. At about which two values of x, between the leftmost and the rightmost low points of $f(x)$ (ignore what's going on beyond these low points), is the graph of $f(x)$ *steepest* (that is, rising or falling most rapidly)? Mark these points of steepest ascent/descent with dots on the above graph.

2. Fill in the blanks below: each blank is to be filled in with one of the following terms (some terms may be used more than once, and some not at all):

   derivative   positive   negative $f(x)$ zero

   To say that the graph of $f(x)$ is at its steepest is to say that the slope, or in other words the ________________, of $f(x)$ is bottoming out (as negative as it can get) or peaking (as _______________ as it can get). But we've seen before that, when a function bottoms out or peaks, its ________________ equals zero. SO: to say that the graph of $f(x)$ is at its steepest is to say that the ________________ of the ________________ of $f(x)$ equals ____________.

3. Find the derivative $f'(x)$ of the function $f(x)$ graphed above, given that

$$f(x) = \frac{x^4}{12} + \frac{x^3}{6} - 3x^2 - 2x + 4.$$

4. For $f(x)$ as above, find $f''(x)$, which is called the *second derivative* of $f(x)$, and means the derivative of the derivative of $f(x)$. (That is, $f''(x) = \frac{d}{dx}[f'(x)]$.)

5. Solve the equation $f''(x) = 0$ for x. Hint: $f''(x)$ factors nicely.

6. Do your results from exercises 1 and 5 above agree? If not, do you want to adjust one of those answers? Please explain.

7. Find $f''(x)$ if

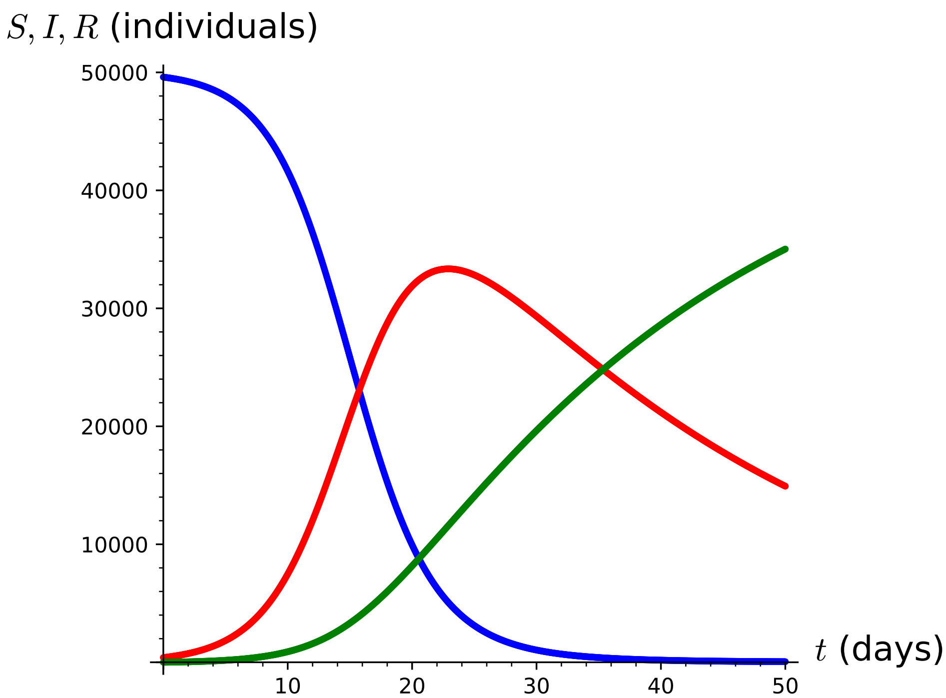
$$f(x) = \sin(x^2).$$

At the end of this exercise, in the space provided, indicate which rule(s) (sum, constant multiple, chain, product, quotient) you used. If you used a rule more than once, state how many times you used it. (You don't have to cite any *formulas* you used, just the rules.)

Rules used: ____

8. Consider a disease evolving according to the usual SIR equations

$$\begin{aligned} S' &= -a\,S\,I, \\ I' &= a\,S\,I - b\,I, \\ R' &= \phantom{-a\,S\,I - {}} b\,I. \end{aligned}$$



Explain carefully why (as the graph suggests) R is at its steepest at the very point where I peaks. Hint: consider differentiating both sides of the equation for R'.