1. Relationship between "new" values, "old" values, and rates of change (also known as "Euler's method"):

$$\text{new } Q = \text{old } Q + \Delta Q \approx \text{old } Q + Q' \cdot \Delta t.$$

2. Definition of the derivative:

$$f'(a) = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}.$$

3. SIR equations:

$$\begin{aligned} S' &= -a\,S\,I, \\ I' &= a\,S\,I - b\,I, \\ R' &= \; b\,I. \end{aligned}$$

4. Threshold value S_T: $S_T = \dfrac{b}{a}$.

5. Equations for lines:

- Slope-intercept form: $y = mx + b$.
- Point-slope form: $y = m(x - x_0) + y_0$.
- Two-point form: $y = m(x - x_1) + y_1$ where $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

6. Derivative formulas:

$$\frac{d}{dx}[c] = 0 \quad (c \text{ a constant}), \quad \frac{d}{dx}x^p = px^{p-1}, \quad \frac{d}{dx}\sin(x) = \cos(x), \quad \frac{d}{dx}\cos(x) = -\sin(x),$$
$$\frac{d}{dx}\tan(x) = \sec^2(x), \quad \frac{d}{dx}e^x = e^x, \quad \frac{d}{dx}b^x = \ln(b)\cdot b^x, \quad \frac{d}{dx}\ln(x) = \frac{1}{x}, \quad \frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}.$$

7. Sum and constant multiple rules:

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x), \quad \frac{d}{dx}[c\,f(x)] = c\,f'(x), \text{where } c \text{ is a constant.}$$

8. Chain rule:

$$\frac{d}{dx}[f(g(x))] = f'(g(x))\,g'(x) \quad \text{or} \quad \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

9. Product and quotient rules:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x), \quad \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

10. Some function properties:

$$\ln(xy) = \ln(x) + \ln(y), \quad \ln(r^s) = s\ln(r), \quad \ln(1/x) = -\ln(x),$$
$$e^{x+y} = e^x e^y, \quad (e^r)^s = e^{rs}, \quad e^{-x} = 1/e^x$$

11. Microscope equation:

$$f(a + \Delta x) \approx f(a) + f'(a)\Delta x.$$

12. Exponential and logistic growth differential equations:

$$\frac{dP}{dt} = kP, \quad \frac{dP}{dt} = kP\left(1 - \frac{P}{b}\right).$$

Here, $k > 0$ is the (per capita) *growth rate*, and $b > 0$ is the *carrying capacity*. The exponential growth DE has solution $P(t) = P_0 e^{kt}$, where P_0 is the initial population.

13. Antiderivative formulas:

$$\int x^p\,dx = \frac{x^{p+1}}{p+1} + C \quad (p \neq -1), \qquad \int \frac{1}{x}\,dx = \ln(|x|) + C, \qquad \int \sin(ax)\,dx = -\frac{\cos(ax)}{a} + C \quad (a \neq 0),$$

$$\int \cos(ax)\,dx = \frac{\sin(ax)}{a} + C \ (a \neq 0), \quad \int e^{ax}\,dx = \frac{e^{ax}}{a} + C \ (a \neq 0), \quad \int b^x\,dx = \frac{b^x}{\ln(b)} + C, \quad \int \frac{1}{1+x^2}\,dx = \arctan(x) + C.$$

14. Integration by substitution:

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du.$$

15. Fundamental Theorem of Calculus:

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a),$$

where $F(x)$ is any antiderivative of $f(x)$.

16. Area of a rectangle = base x height; Area of a triangle = $\frac{1}{2}$(base x height)

17. Relative frequency density (RFD):

$$RFD = \frac{\text{frequency (number of data points in given bin)}}{(\text{bin width } B) \times (\text{data set size } n)}$$

18. Mean and standard deviation:

$$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}, \quad s = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n-1}}.$$

19. Mean and standard deviation for grouped data:

$$\overline{x} = \frac{f_1 y_1 + f_2 y_2 + \cdots + f_m y_m}{n}, \quad s = \sqrt{\frac{f_1(y_1 - \overline{x})^2 + f_2(y_2 - \overline{x})^2 + \cdots + f_m(y_m - \overline{x})^2}{n-1}},$$

where $y_1, y_2, \ldots, y_m$ are the distinct values that the data assumes, and each value y_k occurs f_k times.

20. Standard normal probabilities. If X is $N(0,1)$, then

$$P(-L < x < L) = p,$$

where the "critical value" L corresponds to the probability p as follows:

- $L = 1$: $p = 0.683 = 68.3\%$
- $L = 2$: $p = 0.955 = 95.5\%$
- $L = 3$: $p = 0.997 = 99.7\%$
- $L = 1.96$: $p = 0.950 = 95\%$
- $L = 2.33$: $p = 0.980 = 98\%$
- $L = 2.576$: $p = 0.990 = 99\%$

21. Confidence intervals: a 95%/98%/99% confidence interval for a population mean μ is given by

$$\left(\overline{x} - L\frac{s}{\sqrt{n}}, \overline{x} + L\frac{s}{\sqrt{n}}\right)$$

where L corresponds to the confidence level p as in item **20** above.

22. Hypothesis testing: The "test statistic" or "z-score" for hypothesis testing of a population mean is given by

$$z = \frac{\overline{x} - \mu_0}{(s/\sqrt{n})}.$$

Compare $|z|$ to the appropriate value of L, as given in item **20** above, for a test at the 95%/98%/99% level.

23. NISNID (normal is standard normal in disguise) fact: If X is $N(\mu, \sigma)$, then $Z = \frac{X - \mu}{\sigma}$ is $N(0,1)$.