**Math 1310: CLS SOLUTIONS Tutorial: Modeling with Differential Equations**

1. Two strains of bacteria – *Calcul S. Coli*, denoted $C$, and *Lifus S. Sciencus*, denoted $L$ – are dissolved in a solution of Sprite, denoted $S$. Below, on this page, is a verbal description of the interactions among the two types of bacteria and the Sprite.

   On the *next* page, you will find:

   (a) Space at the top of the page for writing down differential (rate) equations for $C'$, $L'$, and $S'$; and

   (b) A list, at the bottom of the page, of terms that might appear in these differential equations.

   Your task is to *build* (and write down) the differential equations in part (a) out of the terms listed in part (b). Each term may be used *at most* once; some won't be used at all.

   Here is the description of the interactions:

   - $C$ grows logistically, with carrying capacity proportional to the amount of $L$ present.
   - Sprite consumes $C$ at a rate proportional to the amount of $C$ present.
   - A pair of $C$ bacteria can spontaneously join to form an $L$ bacterium; the overall rate at which this occurs is proportional to the number of possible $C$-to-$C$ interactions;
   - $L$ grows at a rate that is, in the absence of other factors, proportional to the amount of $L$ present, but that is inhibited by $S$: the more $S$ there is present, the more slowly $L$ grows;
   - An $L$ bacterium can spontaneously split into two $C$ bacteria; the overall rate at which this occurs is proportional to the amount of $L$ present;
   - $S$ grows at a rate proportional to the amount of $C$ present times the amount of $L$ present;
   - Each *individual* $C$ bacterium consumes $S$ at a rate proportional to the amount of $S$ present;
   - $L$ consumes Sprite at a rate proportional to the amount of $L$ present.

   SOME HINTS:

   - You should need exactly *ten* of the terms from part (b) to construct your differential equations in part (a).
   - "The number of possible $C$-to-$C$ interactions" is proportional to $C^2$, since every $C$ bacterium can interact with every other $C$ bacterium.
   - Since a pair of $C$ bacteria can join to form an $L$ bacterium, the rate at which $C$ bacteria are lost to this process must be twice the rate at which $L$ bacteria are produced by it.

- Similarly, since an $L$ bacterium can split to form a pair of $C$ bacteria, the rate at which $C$ bacteria are produced by this process must be twice the rate at which $L$ bacteria are lost to it it.

(a)

$$C' = +aC\left(1-\frac{C}{bL}\right) - wC - 2\ell C^2 + 2tL$$

$$L' = +\frac{dL}{1+eS} + \ell C^2 - tL$$

$$S' = +gCL - vCS - hL$$

(b) (In the terms below, all lower case letters are positive parameters, and all upper case letters are variables.) Not all terms will be used.

$$+aC\left(1-\frac{CL}{b}\right) \quad ; \quad -eS \quad ; \quad -wC \quad ; \quad +tL \quad ; \quad +\ell C^2 \quad ;$$

$$+2tL \quad ; \quad -tL \quad ; \quad -2\ell C^2 \quad ; \quad +gCL \quad ; \quad +aC\left(1-\frac{C}{bL}\right) \quad ;$$

$$-hL \quad ; \quad -vCS \quad ; \quad +\frac{dL}{1+eS} \quad ; \quad +d(L-eS)$$

2. Chocolate (C), vanilLa (L), and Strawberry (S) ice cream interact in a sundae, according to the differential equations below. By filling in the blanks below, describe their interactions in ways that are consistent with the terms in these differential equations.

$$C' = aCL - \frac{c}{1+dS}$$

$$L' = kL\left(1 - \frac{L}{qS}\right) - fL$$

$$S' = nCS + 3fL$$

Chocolate ice cream grows at a rate ____proportional____ to the product of the amount of Chocolate and ____vanilLa____ ice cream present. Chocolate ice cream melts, but the melting is inhibited by ____Strawberry____ ice cream: the more ____Strawberry____ ice cream, the more slowly ____Chocolate____ ice cream melts.

____vanilLa____ ice cream grows logistically, with ____carrying____ capacity proportional to the amount of ____Strawberry____ ice cream present. Also, a molecule of vanilLa ice cream can spontaneously melt into three molecules of ____Strawberry____ ice cream; this happens at a rate proportional to the amount of ____vanilLa____ ice cream present.

Strawberry ice cream grows at a rate proportional to the product of the amounts of ____Chocolate____ and ____Strawberry____ ice cream present. Strawberry ice cream doesn't melt. (And, again, three molecules of Strawberry ice cream can be spontaneously generated from one molecule of ____vanilLa____ ice cream.)