Week 9 - Thursday, 3/14

Intro to "accumulation functions": power and energy.

Part A: constant power.

Definition. If power is supplied/consumed at a constant rate of p watts (w) for a period of T hours (h), then the energy E supplied/consumed is given by

$$E = pT \qquad (\ast_=)$$

watt-hours (wh).

Notes

(i) We also have kw, mw, kwh, mwh: k for "kilo" (= 1,000) and m for "mega" (= 1,000,000).

(ii) $E = pT$ gives $p = E/T$: power is a rate of energy generation/consumption. (Think of a watt as a "watt-hour per hour.")

(iii) Sometimes energy is measured in joules: 1 joule $= \frac{1}{3600}$ watt-hour (= 1 watt-second).

Example A. A 60 w bulb burning for 2.5 h consumes $60 \cdot 2.5 = 150$ wh of energy.

Part B: power varies "in steps."

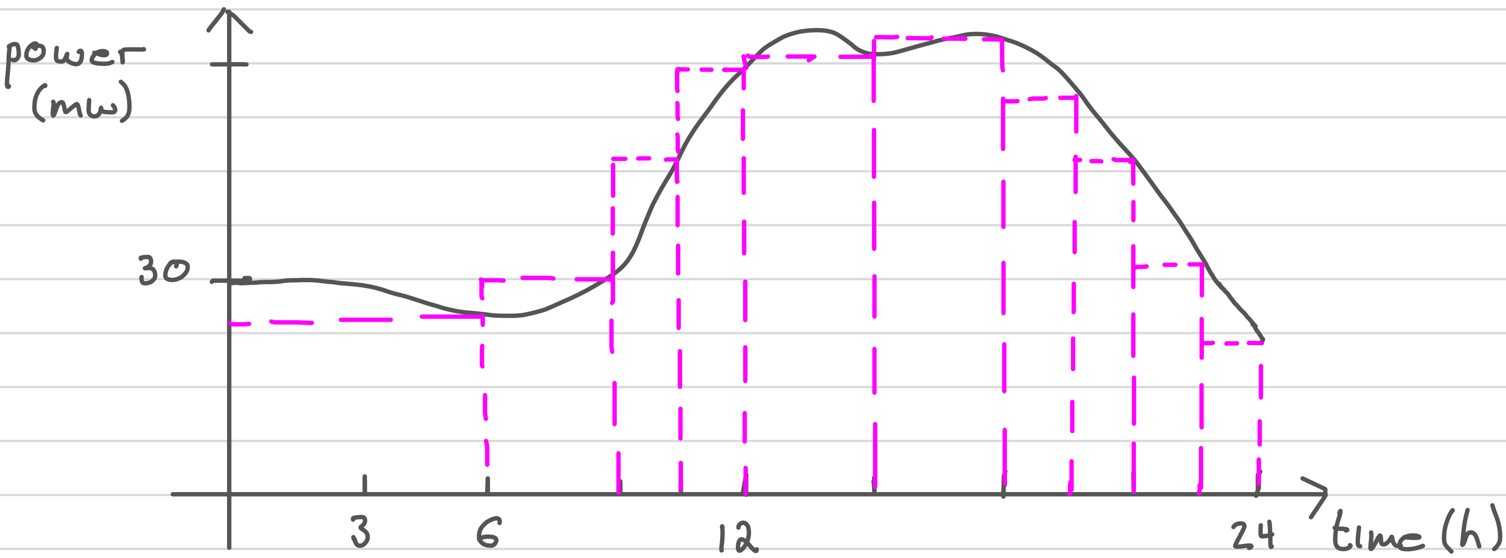
The idea here is to use $(\ast_=)$ for each step, then add.

Example B. If a 3-way bulb burns for 2h at 60w, then for 1.5 h at 100w, then for 3 h at 75w, it consumes

$$E = 2\cdot 60 + 1.5\cdot 100 + 3\cdot 75 = 495 \text{ wh of energy.}$$

Part C: continuously varying power.

Consider this graph of power consumption versus time, in a certain town over a 24-hour period:



We divide [0,24] into subintervals on each of which p doesn't change too much. On each subinterval, we have

$$E \approx pT, \qquad (*_{\approx})$$

where T is the length of the interval, and p is the power value at some point – for this example, we choose the right endpoint – of that interval.

We can now add up, as in part B, to estimate total energy consumption over [0,24].

We find from the graph that:

$E \approx 23 \cdot 6 + 30 \cdot 3 + 47 \cdot 1.5 + 59 \cdot 1.5 + 61 \cdot 3 + 63 \cdot 3 + 55 \cdot 1.5$
$+ 47 \cdot 1.5 + 33 \cdot 1.5 + 22 \cdot 1.5$

$= 994.5$ mwh.

Part D: OK, but where's the calculus?

Let's reconsider/rewrite ($*_{\approx}$): it says the change ΔE in energy used/produced, over an interval of length Δt, is given by

$$\Delta E \approx p(t)\,\Delta t, \qquad (*_{\Delta})$$

if Δt is small and t is any point in the interval. Dividing by Δt gives

$$p(t) \approx \frac{\Delta E}{\Delta t}.$$

We'd expect the "$\approx$" to become "$=$" as Δt shrinks to zero. That is:

$$p(t) = \lim_{\Delta t \to 0} \frac{\Delta E}{\Delta t}:$$

but the right side is, by definition of the derivative, equal to

$$E'(t) \quad !!$$

CONCLUSION:

$E = pT$, for p constant, implies that
$E'(t) = p(t)$
for varying $p(t)$.

Remarks:

(1) As we'll see, the above CONCLUSION holds in many contexts (not just when E = energy and p = power)!

(2) In part C above, we found E by adding up the areas of the pink rectangles. Surprising result: area and derivatives (as in the above CONCLUSION) are related!