Week 9 - Tuesday, 10/20

Hodgkin-Huxley (HH), concluded.

So far, we've seen that:

$$\boxed{c\frac{dV}{dt} = I_E - G_{Na}(V-e_{Na}) - G_K(V-e_K) - G_L(V-e_L)} \quad (6)$$

where:

- c = capacitance of a neural cell membrane;
- V = voltage across the membrane;
- I_E = externally applied current;
- For any channel "Γ = Na (sodium), K (potassium), or L (leakage), we have:
  - G_Γ = conductance of Γ,
  - e_Γ = equilibrium potential of Γ.

Part 3: conductance.

(a) Na:

Each sodium channel comprises four "gates": three "activation gates," each of which has some probability M of being permissive (allowing sodium ions through); and one "inactivation" gate, with some probability H of being permissive.

So the probability of sodium ions flowing through a given sodium channel is M^3H (by probability laws). There are <u>many</u> sodium channels. The conductance G_{Na} thus satisfies

$$\boxed{G_{Na} = g_{Na}M^3H} \quad (7)$$

where g_{Na} is the <u>maximum</u> possible conductance (when all channels are open).

(b) K: similar to Na, but all four gates are of the same type. Call the associated "permissivity probability" N; then

$$\boxed{G_K = g_K N^4.} \quad (8)$$

(c) L: easier, because the conductance G_L is <u>constant</u>. Let's write

$$\boxed{G_L = g_L.} \quad (9)$$

We put (7)(8)(9) into (6) to get

$$\boxed{c\frac{dV}{dt} = I_E - g_{Na}M^3H(V-e_{Na}) - g_K N^4(V-e_K) - g_L(V-e_L).} \quad (10)$$

Part 4: permissivity probabilities.

Consider H, the probability that a sodium inactivation gate is permissive. Such a gate has probability $1-H$ of being impermissive. Say such a gate transitions from permissive to impermissive at a rate A_H, and the reverse way at a rate B_H. Then we find that

$$\boxed{\frac{dH}{dt} = A_H H + B_H(1-H).} \quad (11)$$

Similarly,

$$\boxed{\frac{dM}{dt} = A_M M + B_M(1-M),} \quad (12) \qquad \boxed{\frac{dN}{dt} = A_N N + B_N(1-N)} \quad (13)$$

Equations (10)-(13) <u>are</u> our HH dynamical system! Remark: $A_H, B_H, A_M, B_M, A_N, B_N$ can be found through experiment/theory, and can be expressed as simple functions of V (which, of course, itself depends on t).