Week 9 - Monday, 10/19

The Hodgkin-Huxley (HH) model for action potential in a neuron.

A.L. Hodgkin and A.F. Huxley (1963): Nobel Prize in Medicine for describing "action potential" - that is, a spike in voltage - across a cell membrane (in a giant squid).

Here are their basic ideas.

Part 0: Set-up.

- t denotes time;
- Na, K, L, and Γ stand for sodium, potassium, "leakage," and "generic channel" respectively.
- All other lower case letters denote positive parameters.
- All other upper case letters denote functions of t.

As we'll see, the HH dynamical system comprises four DE's: one for voltage V, and one for each of three "permissivity probabilities" H, M, and N.

Part 1: Voltage and current.

Voltage V is a measure of work required to move a charge against an electric field.

It's related to charge Q and capacitance c (= a measure of charge storage capacity of the membrane) by

$$Q = cV. \quad (1)$$

We define current I to be the rate of change of charge:

$$I = \frac{dQ}{dt} \overset{\text{by (1)}}{=} \frac{d}{dt}[cV] = c\frac{dV}{dt}. \quad (2)$$

In the HH model, I comprises four components:

$$I = I_E - I_{Na} - I_K - I_L, \quad (3)$$

where

I_E = externally applied current,
I_{Na} = sodium ion current through sodium channels,
I_K = potassium ion current through potassium channels,
I_L = leakage current (mostly chloride ions), due to natural permeability of cell membrane.

Combining (2) and (3) gives

$$c\frac{dV}{dt} = I_E - I_{Na} - I_K - I_L. \quad (4)$$

Part 2: Ohm's Law.

Ohm's Law says that the current I_Γ through any of the above channels Γ satisfies

$$I_\Gamma = G_\Gamma(V - e_\Gamma) \quad (5)$$

where G_Γ is the conductance (a measure of compliance to current) of the channel, and e_Γ is the channel's equilibrium potential.

Applying (5), with $\Gamma = Na, K$, and L, to (4) gives

$$c\frac{dV}{dt} = I_E - G_{Na}(V - e_{Na}) - G_K(V - e_K) - G_L(V - e_L). \quad (6)$$

To be continued.