Week 9 - Friday, 10/23

Intro to "accumulation functions": power and energy.

Part A: constant power.

Definition. If power is supplied/consumed at a constant rate of p watts (w) for a period of Δt hours (h), then the net energy ΔE supplied/consumed is given by

$$\Delta E = p\,\Delta t \qquad (*=)$$

watt-hours (wh).

Notes

(i) We also have kw, mw, kwh, mwh: k for "kilo" (= 1,000) and m for "mega" (= 1,000,000).

(ii) $\Delta E = p\,\Delta t$ gives $p = \frac{\Delta E}{\Delta t}$: power is a rate of energy generation/consumption. (Think of a watt as a "watt-hour per hour".)

(iii) Sometimes energy is measured in joules: 1 joule $= \frac{1}{3600}$ watt-hour (= 1 watt-second).

Example A. A 60 w bulb burning for 2.5 h consumes
$$\Delta E = 60 \cdot 2.5 = 150 \text{ wh of energy.}$$

Part B: power varies "in steps."

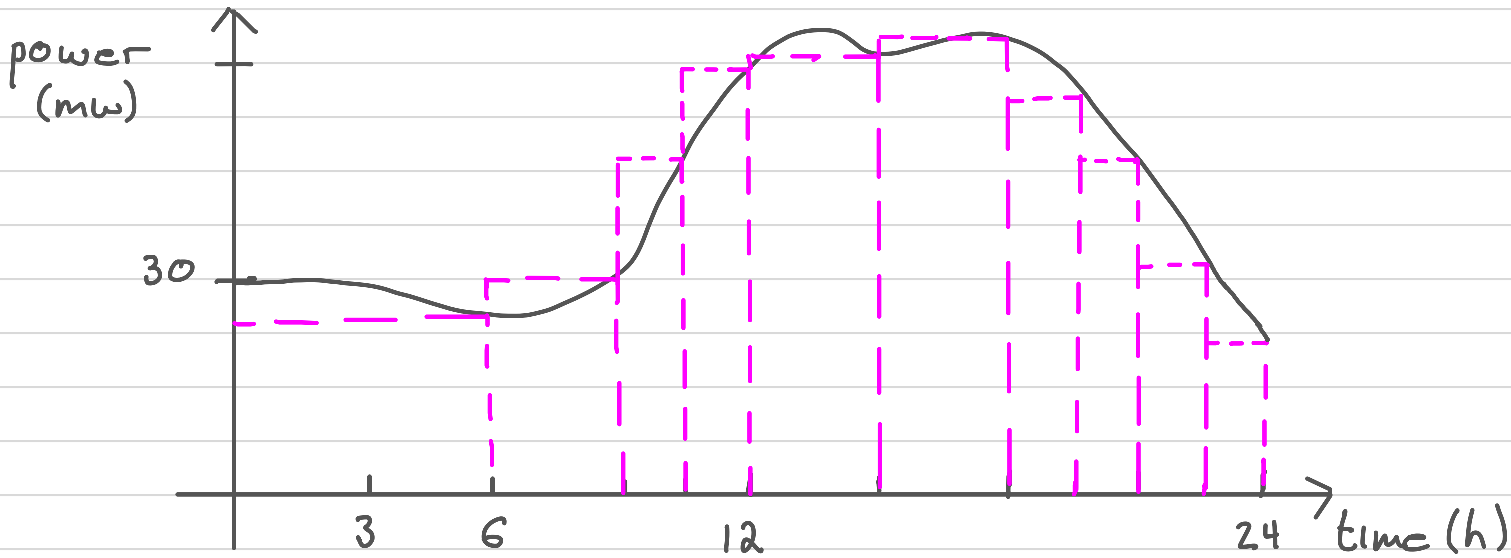
The idea here is to use $(*=)$ for each step, then add.

Example B. If a 3-way bulb burns for 2h at 60w, then for 1.5 h at 100w, then for 3 h at 75w, it consumes

$\Delta E = 2 \cdot 60 + 1.5 \cdot 100 + 3 \cdot 75 = 495$ wh of energy.

<u>Part C:</u> continuously varying power.

Consider this graph of power consumption versus time, in a certain town over a 24-hour period:



We divide $[0, 24]$ into <u>subintervals</u> on each of which p doesn't <u>change</u> too much. On each subinterval, we have

$$\Delta E \approx p \, \Delta t, \qquad (*_{\approx})$$

where Δt is the length of the interval, and p is the power value at some point - for this example, we choose the right endpoint - of that interval.

We can now <u>add up</u>, as in part B, to <u>estimate</u> total energy consumption over $[0, 24]$.

We find from the graph that:

$$\Delta E \approx 23\cdot 6 + 30\cdot 3 + 47\cdot 1.5 + 59\cdot 1.5 + 61\cdot 3 + 63\cdot 3 + 55\cdot 1.5 + 47\cdot 1.5 + 33\cdot 1.5 + 22\cdot 1.5$$

$$= 994.5 \text{ mwh.}$$

Remark:

Had we divided $[0,24]$ into even narrower subintervals, we would most likely get a better estimate for ΔE.

Food for thought:

can we get an exact answer by breaking $[0,24]$ into infinitely many, infinitesimally narrow subintervals?