Week 8 - Monday, 3/4

Modeling with initial value problems - an overview.

BIG IDEA: many natural phenomena can be understood in terms of their rates of change (= derivatives).

Specifically: many phenomena can be modeled by initial value problems (IVP's), consisting of:

(a) One or more differential equations (DE's), describing the rates of change, together with

(b) One or more initial conditions (IC's), describing how things look "at the outset" (or at some particular point).

EXAMPLES.

(1) The SIR epidemiology model consists of

(a) DE's

$$\left.\begin{aligned} S' &= -aSI \\ I' &= aSI - bI \\ R' &= bI \end{aligned}\right]$$

the parameters a and b must be specified

and

(b) IC's

$$S(0) = S_0, \; I(0) = I_0, \; R(0) = R_0.$$

(2) In an exponential growth/decay situation, we have a DE

$$\frac{dP}{dt} = kP \quad \text{or} \quad \frac{dR}{dt} = -kR$$

and an IC

$$P(0) = P_0 \quad \text{or} \quad R(0) = R_0.$$

BIG QUESTION:

How do we solve an IVP? That is: from info about rates of change and starting values, how do we deduce explicit info (formulas, graphs, values, etc.) about the quantities themselves?

There are, generally, two ways:

(A) Analytic, or closed-form, solutions.

If we're lucky, we can express the dependent variables directly (by formulas) in terms of the independent variable.

For example, we've seen that the IVP

$$\frac{dP}{dt} = kP, \qquad P(0) = P_0$$

has closed form solution $P(t) = P_0 e^{kt}$.

More on closed form solutions later.

(B) Numerical (approximate) solutions.

Closed form solutions can be difficult or impossible to obtain. In such cases, we can use "Euler's method," meaning the idea that

$$\begin{aligned} \text{new } Q &= \text{old } Q + \Delta Q \\ &\approx \text{old } Q + Q' \cdot \Delta t. \end{aligned}$$

Together with software like Sage, we can then approximate successive values of our quantities through iteration:

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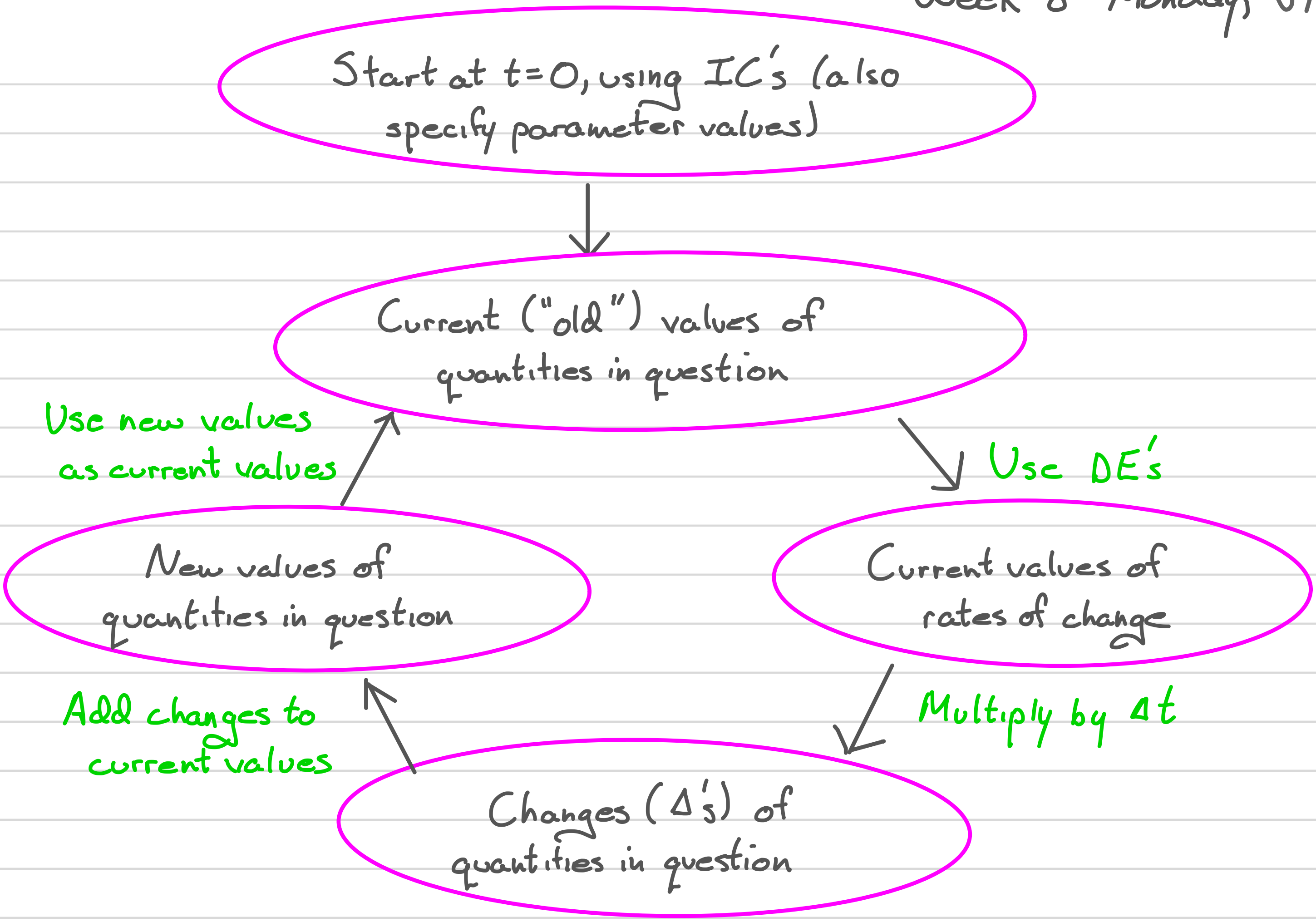

(Stop at desired t value.)

<u>SUMMARY</u>. IVP's model many natural phenomena. IVP's can be solved:

(A) In closed form, meaning we find solutions like

$Q(t)$ = some explicit function of the independent variable t

for each quantity Q; or

(B) Numerically, meaning we use *Euler's* (or a similar) method to generate lists

$Q(0), Q(\Delta t), Q(2\Delta t), Q(3\Delta t), \ldots$

of (approximate) values for each of the quantities Q in question.