Week 8 - Tuesday, 10/13

Modeling with IVP's: examples.
Today: population growth.

(A) Exponential (unlimited) growth

Here, the rate of growth of the population is proportional to the population size:

$$\frac{dP}{dt} = kP, \qquad P(0) = P_0. \qquad (*)$$

We've seen that $(*)$ has closed-form solution

$$P(t) = P_0 e^{kt}.$$

(B) Logistic (bounded) growth.

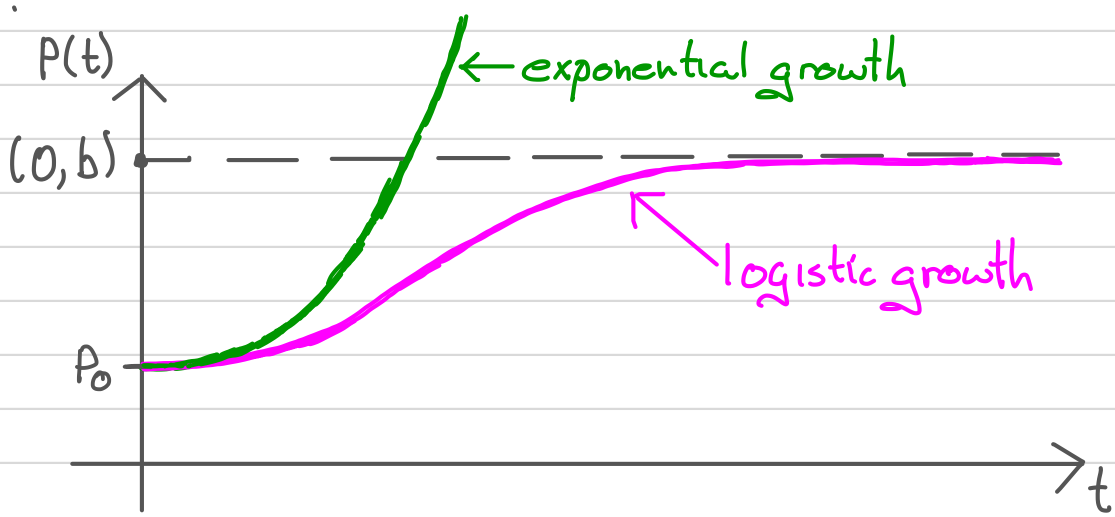
If an environment can support only a finite number - say, b - individuals, then instead of $(*)$, we might have

"per capita growth rate"

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{b}\right), \qquad P(0) = P_0. \qquad (*')$$

"carrying capacity"

Picture:



One can show that (*') has closed-form solution

$$P(t) = \frac{P_0 b e^{kt}}{b + P_0(e^{kt} - 1)}.$$

We've also studied (*') using Euler's method.

(c) Dual populations, e.g. predator/prey.

Say we have rabbits R and foxes F. We assume:

(i) exponential growth of R, in the absence of F;

(ii) rabbits die at a rate proportional to $R \cdot F$;

(iii) Foxes are born at a rate proportional to $R \cdot F$;

(iv) Foxes die at a rate proportional to F.

These assumptions yield the "Lotka-Volterra equations:"

$$(LV) \begin{cases} \dfrac{dR}{dt} = \overbrace{aR}^{(i)} - \overbrace{bRF}^{(ii)} \\[2ex] \dfrac{dF}{dt} = \underbrace{cRF}_{(iii)} - \underbrace{dF}_{(iv)} \end{cases}$$

(a, b, c, d are positive parameters).

There are no closed-form solutions to (LV); we need to use numerical methods to solve them.