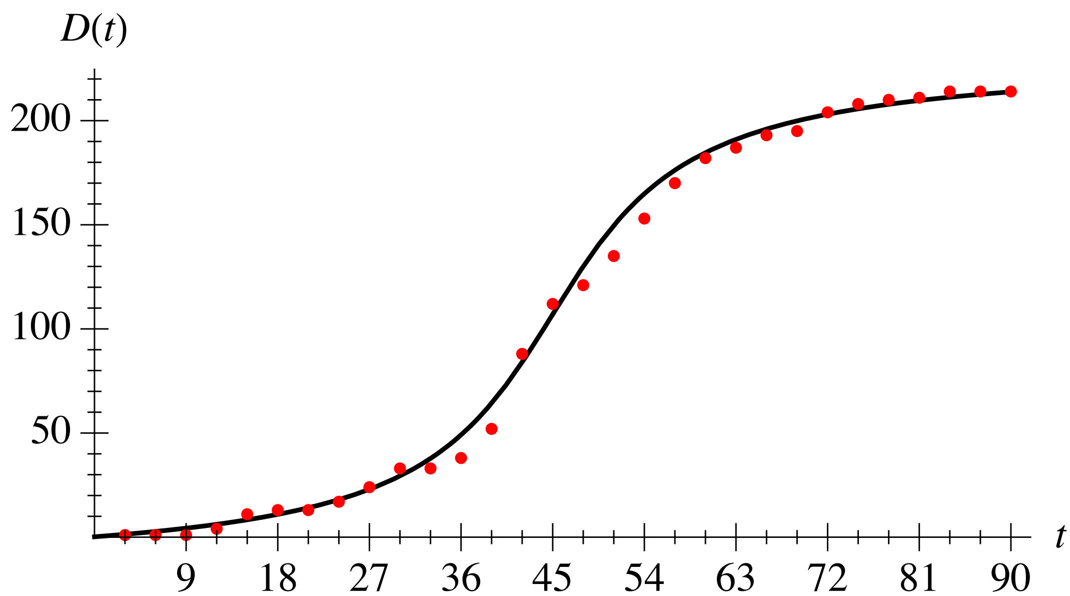
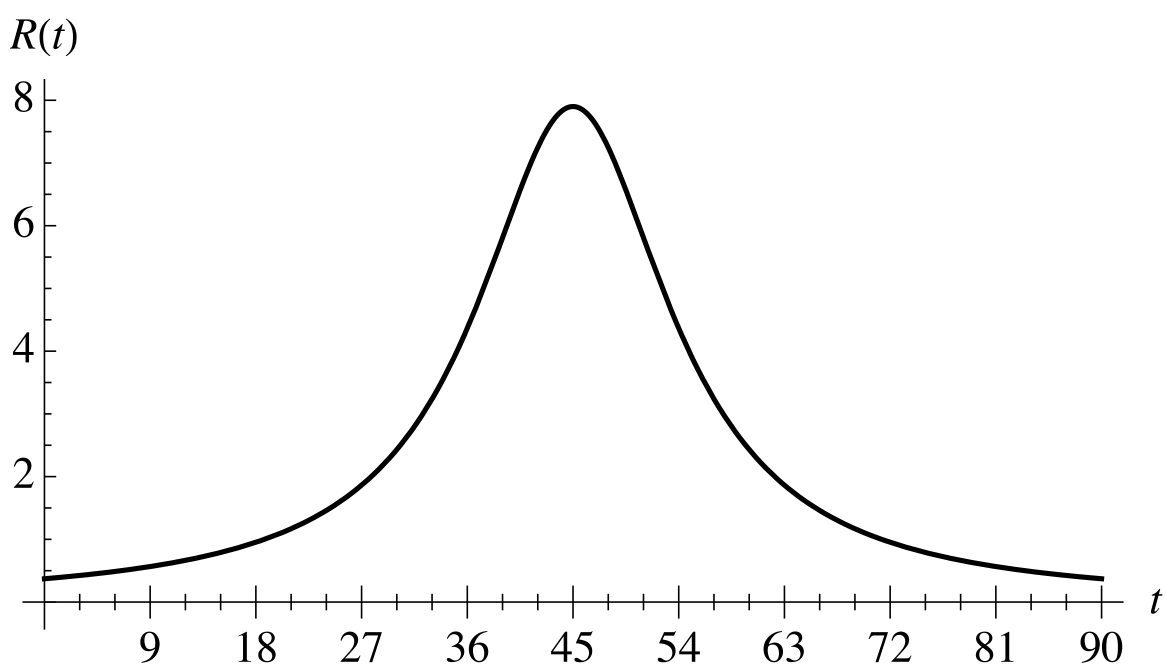
The graph below shows a function $D(t)$ given by the formula

$$D(t) = 79\arctan(0.1(t-45)) + 107.$$



The next graph shows a function $R(t)$ given by the formula

$$R(t) = \frac{7.9}{1+0.01(t-45)^2}.$$



1. Let $D(t)$ and $R(t)$ be the functions given on the previous page. Use the formula

$$\frac{d}{dt}[\arctan(t)] = \frac{1}{1+t^2},$$

together with any relevant differentiation rules and formulas, to show that

$$D'(t) = R(t).$$

$$\begin{aligned}
D'(t) &= \frac{d}{dt}\left[79\arctan(0.1(t-45)) + 107\right] \\
&= 79\frac{d}{dt}\left[\arctan(0.1(t-45))\right] + \frac{d}{dt}[107] \\
&= 79\frac{1}{1+(0.1(t-45))^2}\cdot\frac{d}{dt}[0.1(t-45))] + 0 \\
&= 79\frac{1}{1+(0.1(t-45))^2}\cdot 0.1 = \frac{7.9}{1+0.01(t-45)^2} \\
&= R(t).
\end{aligned}$$

We are now going to examine data from an actual 90-day-long outbreak of Ebola in the Democratic Republic of Congo (DRC), in 1995.

Here is a table of deaths due to this outbreak. The top row of the table breaks up the ninety-day period into three-day increments. The bottom row denotes the total, or *cumulative*, number of deaths $C(t)$, due to this Ebola outbreak, from day 0 to the end of day t.

Time t (days)	3	6	9	12	15	18	21	24	27	30
Total deaths $C(t)$	1	1	1	4	11	13	13	17	24	33

Time t (days)	33	36	39	42	45	48	51	54	57	60
Total deaths $C(t)$	33	38	52	88	112	121	135	153	170	182

Time t (days)	63	66	69	72	75	78	81	84	87	90
Total deaths $C(t)$	187	193	195	204	208	210	211	214	214	214

2. Directly on the *top* graph on the first page of this worksheet, plot the points given in the above table. That is: plot (with a small circle or dot) each of the points $(t, C(t))$, where $t = 3, 6, 9, \ldots, 90$. (You don't need to connect the dots.) See graph above.

3. Do you now believe that the arctangent function has relevance to the study of disease? Justify your answer.

You bet I do. The actual cumulative death data points $(t, C(t))$ conform quite closely to the graph of $D(t)$, which *is* a modified arctangent function. So an arctangent function does seem to model this particular Ebola outbreak pretty well.

4. Consider the Ebola outbreak documented in the above table, on the previous page. Instead of looking at *cumulative* deaths, let's now consider the *death rate*, measured in deaths per day. What do you think the graph of the death rate function might look like? Please explain.

Death rate is the rate of change of cumulative deaths. So the death rate function should look like the derivative of the cumulative deaths function $C(t)$. But $C(t)$ looks like $D(t)$, and the derivative of $D(t)$ is the curve $R(t)$ appearing in the bottom graph of the first page. In sum: the death rate function should look something like $R(t)$.

5. Fill in the blanks – in each blank *except for the last one*, the correct answer is *either* the word "sigmoid" (which means "roughly S-shaped") or the word "bell."

The cumulative death function for the above Ebola outbreak can be modeled fairly well by an arctangent curve, which has something of a ____sigmoid____ shape. The death rate for the outbreak can then be modeled fairly well by a curve that has something of a ____bell____ shape.

OK, so, the derivative of a ____sigmoid____ curve is a ____bell____ curve. Where have we seen this before? We've seen it in SIR!! Remember that, there, the variable R (recovered) followed a ____sigmoid____ curve, while the variable I (infected) followed a ____bell____ curve. But I *is* the derivative of R, or more precisely I is *proportional to* the derivative of R, by the third of the SIR equations, which says $R' =$ ____bI____.