1. Find the following derivatives.

   (a) $\dfrac{d}{dx}\left[5\arctan(3x)\right]$

$$\frac{d}{dx}\left[5\arctan(3x)\right] = \frac{15}{1+9x^2}.$$

   (b) $\dfrac{d}{dx}\left[\arctan(x^3) + \arctan^3(x)\right]$.

$$\frac{d}{dx}\left[\arctan(x^3) + \arctan^3(x)\right] = \frac{3x^2}{1+x^6} + \frac{3\arctan^2(x)}{1+x^2}.$$

2. Let $f(x) = \arctan(x^2)$. Show that

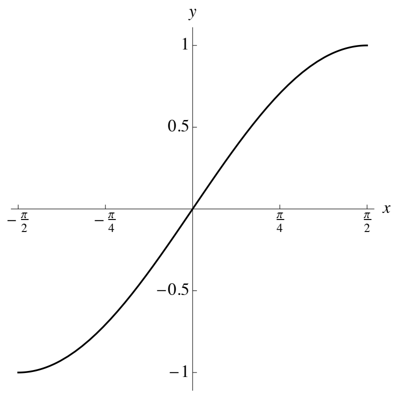
$$f''(x) = \frac{2 - 6x^4}{(1 + x^4)^2}.$$

$$\begin{aligned} f'(x) &= \frac{d}{dx}[\arctan(x^2)] = \frac{1}{1 + \left(x^2\right)^2} \frac{d}{dx}[x^2] \\ &= \frac{2x}{1 + x^4}. \end{aligned}$$

So

$$\begin{aligned} f''(x) &= \frac{d}{dx}\left[\frac{2x}{1 + x^4}\right] = \frac{(1 + x^4)\frac{d}{dx}[2x] - 2x\frac{d}{dx}[1 + x^4]}{\left(1 + x^4\right)^2} \\ &= \frac{(1 + x^4) \cdot 2 - 2x \cdot 4x^3}{\left(1 + x^4\right)^2} = \frac{2 + 2x^4 - 8x^4}{\left(1 + x^4\right)^2} \\ &= \frac{2 - 6x^4}{\left(1 + x^4\right)^2}. \end{aligned}$$

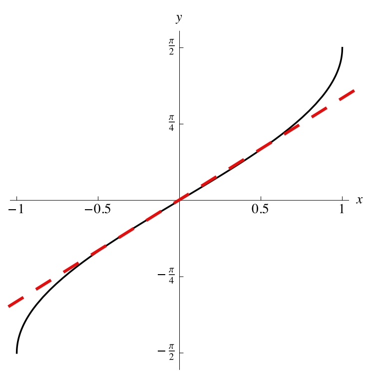
3. On the axes below is a graph of the function $y = \sin(x)$, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.



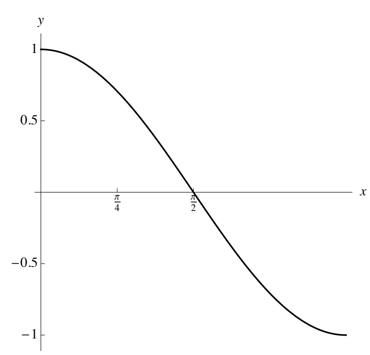
(a) Explain why reflecting $y = \sin(x)$, on this domain, about the line $y = x$ gives a new function, which we'll call $y = \arcsin(x)$.

The function $y = \sin(x)$, on the given domain, satisfies the horizontal line test, so its reflection about $y = x$ satisfies the vertical line test, and is therefore a function.

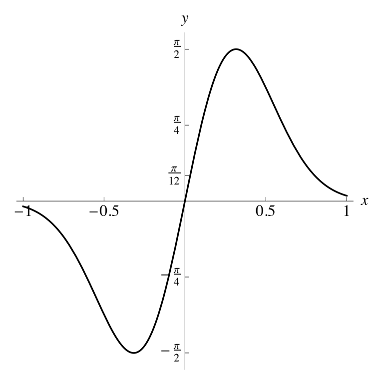
(b) Which of the following gives the graph of $y = \arcsin(x)$? Circle the letter ((A), (B), or (C)) below the correct graph.



(A)



(B)



(C)

(c) Find $\frac{d}{dx}[\arcsin(x)]$, as follows (fill in the blanks; there are six of them).

Since the function $y = \arcsin(x)$ takes an input x to an output $\arcsin(x)$, we know that the reflection $y = \sin(x)$ must take an input $\arcsin(x)$ to an ____output____ x. That is,

$$\sin(\arcsin(x)) = \underline{\quad x \quad}. \tag{1}$$

We differentiate both sides of this equation to get

$$\frac{d}{dx}[\sin(\arcsin(x))] = 1$$

or, using the chain rule on the left,

$$\cos(\underline{\quad \arcsin(x) \quad}) \cdot \frac{d}{dx}[\underline{\quad \arcsin(x) \quad}] = 1.$$

Dividing by $\cos(\arcsin(x))$ then gives

$$\frac{d}{dx}[\arcsin(x)] = \frac{1}{\cos(\arcsin(x))}. \tag{2}$$

Now for any real number θ, we have $\cos(\theta) = \sqrt{1 - (\sin(\theta))^2}$. But then

$$\cos(\arcsin(x)) = \sqrt{1 - (\sin(\arcsin(x)))^2} = \sqrt{1 - (\underline{\quad x \quad})^2},$$

the last step by equation (1). Putting this back into equation (2) gives

$$\frac{d}{dx}[\arcsin(x)] = \underline{\quad \frac{1}{\sqrt{1-x^2}} \quad},$$

and we're done.

(d) Find the equation of the line tangent to the graph of $f(x) = \arcsin(x)$ at $x = 0$. Draw this tangent line on the arcsin graph you chose in part (b) of this exercise.

$f(0) = \arcsin(0) = 0$, and $f'(0) = \frac{1}{\sqrt{1-0^2}} = 1$. So the equation is

$$y = 1(x - 0) + 0 = x.$$