Week 7 - Monday, 10/5

The derivative of ln(x).

Last week, we saw that

$$\boxed{\ln(e^x) = x} \qquad (*)$$

for any real number x, and

$$\boxed{e^{\ln(x)} = x} \qquad (*')$$

for any $x > 0$.

Take equation $(*')$ and differentiate both sides:

$$\frac{d}{dx}\left[e^{\ln(x)}\right] = \frac{d}{dx}[x].$$

Applying the chain rule on the left and a simple formula on the right, we get

$$e^{\ln(x)} \frac{d}{dx}[\ln(x)] = 1.$$

Divide by $e^{\ln(x)}$:

$$\frac{d}{dx}[\ln(x)] = \frac{1}{e^{\ln(x)}}.$$

Finally, apply $(*')$ to the right, to get

$$\boxed{\frac{d}{dx}[\ln(x)] = \frac{1}{x}.}$$

Derivative of the natural logarithm function.

Examples. Find

(a) $\frac{d}{dx}[\cos(1-\ln(x))]$, (b) $\frac{d}{dx}[\ln(1-\cos(x))]$, (c) $\frac{d}{dx}[\ln(e^x)]$.

Solution.

(a) $\frac{d}{dx}[\cos(1-\ln(x))] = -\sin(1-\ln(x)) \cdot \frac{d}{dx}[1-\ln(x)]$

$= -\sin(1-\ln(x)) \cdot (0 - 1/x) = \frac{\sin(1-\ln(x))}{x}$.

(b) $\frac{d}{dx}[\ln(1-\cos(x))] = \frac{1}{1-\cos(x)} \cdot \frac{d}{dx}[1-\cos(x)] = \frac{\sin(x)}{1-\cos(x)}$.

(c) $\frac{d}{dx}[\ln(e^x)] = \frac{1}{e^x} \cdot \frac{d}{dx}[e^x] = \frac{1}{e^x} \cdot e^x = 1$.

(Note: we can also simplify first:

$\frac{d}{dx}[\ln(e^x)] \underset{\text{by } (*')}{=} \frac{d}{dx}[x] = 1.$)