Week 6 - Friday, 10/2

The natural logarithm function.

I) Graph.

Take the function $y=e^x$ and swap the roles of x and y. Then:

(a) Geometrically, we are swapping the horizontal and vertical axes, which amounts to a reflection about the line $y=x$.

(b) Algebraically, we are writing

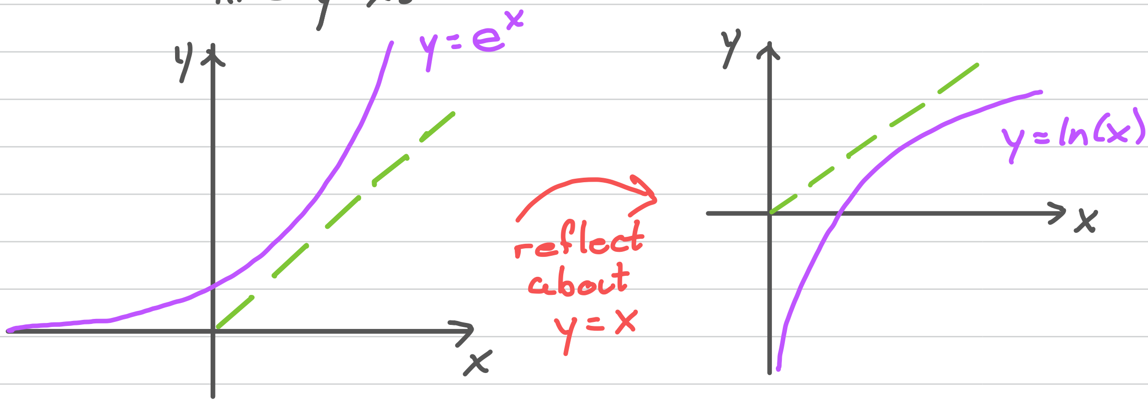
$$x=e^y.$$

Note we can solve for y using $\ln$:

$$\ln(x)=\ln(e^y)$$
$$\ln(x)=y.$$

CONCLUSION:

$y=\ln(x)$ is $y=e^x$, reflected about the line $y=x$!



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II) Properties of $\ln(x)$.

These all come from properties of e^x and the equation

$$\ln(e^a) = a. \qquad (*)$$

For example, $(*)$ gives $\ln(e^0) = 0$. But $e^0 = 1$. So:

(a) $\ln(1) = 0$. Some others:

(b) $\ln(e) = 1$.

(c) $\ln(1/x) = -\ln(x)$ for $x > 0$.

(d) $\ln(xy) = \ln(x) + \ln(y)$ for $x, y > 0$.

(e) $\ln(x/y) = \ln(x) - \ln(y)$ for $x, y > 0$.

(f) $\ln(r^s) = s\ln(r)$ for $r > 0$.

<u>Example 1.</u>

Simplify

$$\ln\left(\frac{a^{10}e^{5x-3}}{x^{\cos(b)}}\right).$$

<u>Solution.</u>

By properties (d)(e)(f), and by $(*)$,

$$\begin{aligned}\ln\left(\frac{a^{10}e^{5x-3}}{b^{\cos(x)}}\right) &= \ln(a^{10}e^{5x-3}) - \ln(x^{\cos(b)}) \\ &= \ln(a^{10}) + \ln(e^{5x-3}) - \ln(x^{\cos(b)}) \\ &= 10\ln(a) + 5x - 3 - \cos(b)\ln(x).\end{aligned}$$

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One more trick: let's raise e to both sides

of $(*)$. We get
$$e^{\ln(e^a)} = e^a. \qquad (🐟)$$

Now e^a could be anything (positive), so call it b. Then equation (🐟) gives
$$\boxed{e^{\ln(b)} = b} \quad (b > 0). \qquad (*')$$

(Together, $(*)$ and $(*')$ say: $y = e^x$ and $y = \ln(x)$ undo each other in <u>either</u> <u>order</u>.)

<u>Example 2.</u> Simplify $e^{x\ln(29) + 5\ln(\cos(x))}$.

<u>Solution.</u>
$$\begin{aligned} e^{x\ln(29) + 5\ln(\cos(x))} &= e^{x\ln(29)} e^{5\ln(\cos(x))} \\ &= \left(e^{\ln(29)}\right)^x \left(e^{\ln(\cos(x))}\right)^5 \\ &= 29^x (\cos(x))^5 \\ &= 29^x \cos^5(x). \end{aligned}$$