Goal: To practice using *both* versions of the chain rule

1. **Chain rule version 1.** Express each of the given functions of x in the form

$$y = f(u) \text{ where } u = g(x).$$

Then use the chain rule version 1:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx},$$

to differentiate.

Example. Find $\frac{dy}{dx}$ if $y = 4^{\sin(x)}$.

SOLUTION. $y = 4^u$ where $u = \sin(x)$. So

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{d}{du}[4^u] \cdot \frac{d}{dx}[\sin(x)] \\ &= \ln(4)\, 4^u \cdot \cos(x) = \ln(4)\, 4^{\sin(x)} \cos(x).\end{aligned}$$

(a) Find $\frac{dy}{dx}$ if $y = \sin(4^x)$.

Write $y = \sin(u)$ where $u = 4^x$. Then

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{d}{du}[\sin(u)] \cdot \frac{d}{dx}[4^x] \\ &= \cos(u) \cdot \ln(4)\, 4^x = \ln(4)\, 4^x \cos(4^x).\end{aligned}$$

(b) Find $\frac{dz}{dq}$ if $z = \tan(7q^2)$.

Write $z = \tan(u)$ where $u = 7q^2$. Then

$$\begin{aligned}\frac{dz}{dq} &= \frac{dz}{du} \cdot \frac{du}{dq} \\ &= \frac{d}{du}[\tan(u)] \cdot \frac{d}{dq}[7q^2] \\ &= \sec^2(u) \cdot 14q = 14q \sec^2(7q^2).\end{aligned}$$

(c) Find y' if $y = 3^{4^x}$. (Note that 3^{4^x} means $3^{(4^x)}$, not $(3^4)^x$.)

Write $y = 3^u$ where $u = 4^x$. Then

$$\begin{aligned} y' = \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{d}{du}[3^u] \cdot \frac{d}{dx}[4^x] \\ &= \ln(3)3^u \cdot \ln(4)4^x = \ln(3)\ln(4)3^{4^x}4^x. \end{aligned}$$

(d) Find y' if $y = 3^{x^4}$.

Write $y = 3^u$ where $u = x^4$. Then

$$\begin{aligned} y' = \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{d}{du}[3^u] \cdot \frac{d}{dx}[x^4] \\ &= \ln(3)3^u \cdot 4x^3 = 4\ln(3)x^3 3^{x^4}. \end{aligned}$$

2. **Chain rule version 2.** Use the chain rule version 2:

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x),$$

to differentiate.

Example. Find

$$\frac{d}{dx}\left[4^{\sin(x)}\right].$$

SOLUTION.

$$\begin{aligned}\frac{d}{dx}\left[4^{\sin(x)}\right] &= \ln(4)\,4^{\sin(x)} \cdot \frac{d}{dx}[\sin(x)]\\ &= \ln(4)\,4^{\sin(x)}\cos(x).\end{aligned}$$

(a) Find $\frac{dy}{dx}$ if $y = \sin(4^x)$.

$$\begin{aligned}\frac{dy}{dx} &= \cos(4^x) \cdot \frac{d}{dx}[4^x]\\ &= \cos(4^x) \cdot \ln(4)\,4^x = \ln(4)\,4^x\cos(4^x).\end{aligned}$$

(b) Find $\frac{dz}{dq}$ if $z = \tan(7q^2)$.

$$\begin{aligned}\frac{dz}{dq} &= \sec^2(7q^2) \cdot \frac{d}{dq}[7q^2]\\ &= \sec^2(7q^2) \cdot 14q = 14q\sec^2(7q^2).\end{aligned}$$

(c) Find y' if $y = 3^{4^x}$.

$$\begin{aligned} y' &= \frac{d}{dx}[3^{4^x}] = \\ &= \ln(3)3^{4^x} \cdot \frac{d}{dx}[4^x] = \ln(3)3^{4^x} \cdot \ln(4)4^x \\ &= \ln(3)\ln(4)3^{4^x}4^x. \end{aligned}$$

(d) Find $\frac{d}{dx}\left[3^{4^{5^x}}\right]$. (Note that $3^{4^{5^x}}$ means $3^{\left(4^{(5^x)}\right)}$.)

$$\begin{aligned} \frac{d}{dx}\left[3^{4^{5^x}}\right] &= \ln(3)3^{4^{5^x}} \cdot \frac{d}{dx}\left[4^{5^x}\right] \\ &= \ln(3)3^{4^{5^x}} \cdot \ln(4)4^{5^x} \cdot \frac{d}{dx}\left[5^x\right] \\ &= \ln(3)3^{4^{5^x}} \cdot \ln(4)4^{5^x} \cdot \ln(55^x \\ &= \ln(3)\ln(4)\ln(5)3^{4^{5^x}}4^{5^x}5^x. \end{aligned}$$