1. **(Derivation of the Microscope Equation.)** Fill in the blanks: Recall that, if the function $f(x)$ is locally ____linear____ at $x = a$, then

$$f'(a) = \lim_{\Delta x \to 0} \frac{f(\underline{\quad a + \Delta x \quad}) - f(a)}{\Delta x}.$$

This means: for such a function, if Δx is small, then

$$\underline{\quad f'(a) \quad} \approx \frac{f(a + \Delta x) - f(a)}{\Delta x}.$$

Multiplying through by Δx gives

$$f'(a)\ \underline{\quad \Delta x \quad} \approx f(a + \Delta x) - f(a).$$

Solving this for $f(a + \Delta x)$, we find that, if Δx is small, then

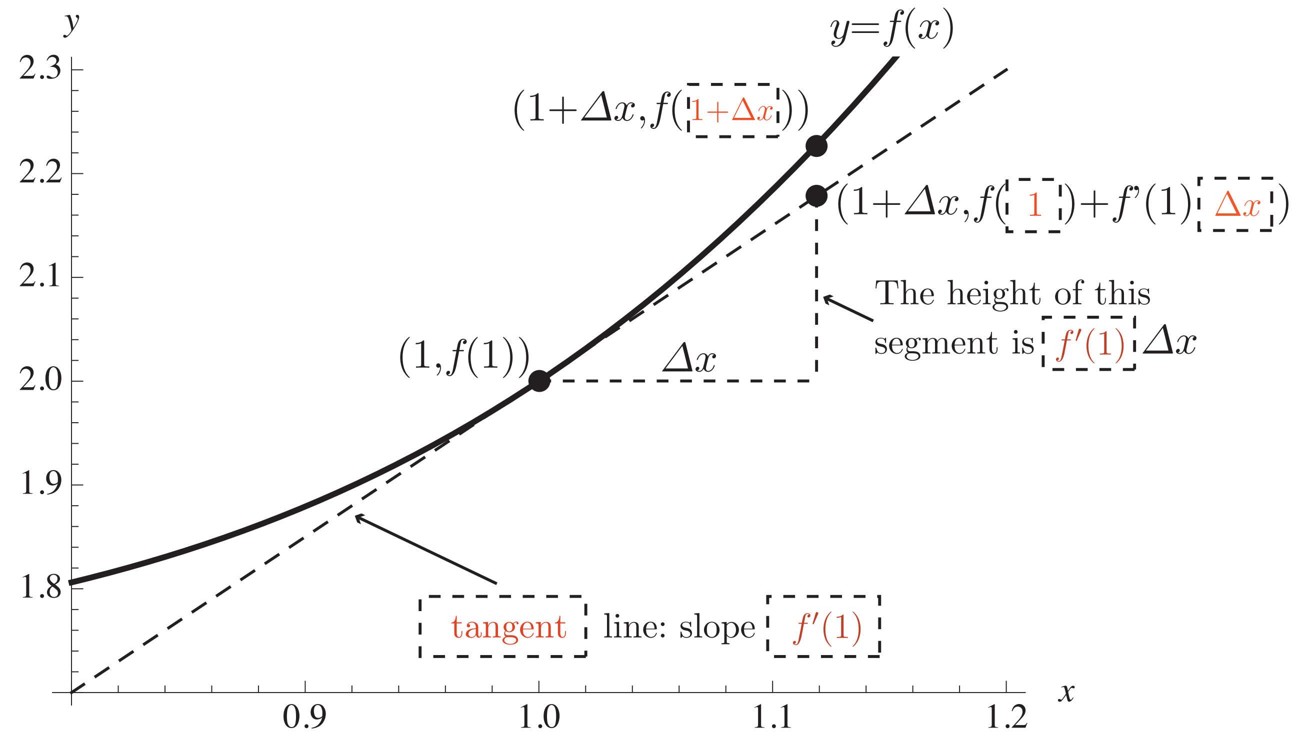
$$f(a + \Delta x) \approx f(a) + \underline{\quad f'(a)\Delta x \quad}.$$

2. **(Geometric interpretation of the Microscope Equation.)** The graph of a function $f(x)$ appears below (solid), along with the graph of the line tangent to $f(x)$ at the point $(1, f(1))$ (dashed). (Note that the axes here do NOT intersect at (0,0).)

   Fill in the dashed rectangles (there are seven of them) on the picture below, by inserting exactly one of the following terms in each rectangle.

   1 Δx $1{+}\Delta x$ $f'(1)$ $f(1){+}f'(1)\Delta x$ tangent

   Note: All terms will be used, some perhaps more than once.



As the picture illustrates, for Δx small,

$$f(1+\Delta x) \approx \boxed{f(1){+}f'(1)\Delta x}$$

3. **(The Chain Rule and the Microscope Equation.)**

(a) Find $\frac{d}{dx}\left[\sqrt{3+\sqrt{x}}\right]$.

$$\frac{d}{dx}\left[\sqrt{3+\sqrt{x}}\right] = \frac{1}{2}\left(3+\sqrt{x}\right)^{-1/2}\frac{d}{dx}\left[3+\sqrt{x}\right] = \frac{1}{2}\left(3+\sqrt{x}\right)^{-1/2}\cdot\frac{1}{2}x^{-1/2}$$
$$= \frac{1}{4\sqrt{x}\sqrt{3+\sqrt{x}}} \quad (\text{or } \frac{1}{4}x^{-1/2}(3+x^{1/2})^{-1/2}).$$

(b) Write down the microscope equation for $f(x) = \sqrt{3+\sqrt{x}}$ at $x = 1$.

We have

$$f(x) = \sqrt{3+\sqrt{x}} \qquad f'(x) = 1/(4\sqrt{x}\sqrt{3+\sqrt{x}})$$
$$a = 1$$
$$f(a) = f(1) = \sqrt{3+\sqrt{1}} = \sqrt{4} = 2 \qquad f'(a) = f'(1) = 1/(4\sqrt{1}\sqrt{3+\sqrt{1}}) = 1/8$$

So the microscope equation, in this case, reads

$$f(a+\Delta x) \approx f(a) + f'(a)\Delta x$$
$$f(1+\Delta x) \approx 2 + \frac{1}{8}\Delta x$$
$$\sqrt{3+\sqrt{1+\Delta x}} \approx 2 + \frac{1}{8}\Delta x.$$

(c) Use your answer to part (b) above to estimate $\sqrt{3+\sqrt{1.04}}$.

We plug $\Delta x = 0.04$ into (b), to get

$$\sqrt{3+\sqrt{1.04}} \approx 2 + \frac{1}{8}(0.04) = 2.005.$$

(The "true" value is 2.00494...)

4. **(Differentiation rules.)** Fill in each blank on the right hand side with the name of the rule that is being used there. (At most one rule is being used per line. In "real life," you might want to combine some steps.) If *no* rules are being used (for example, if only a differentiation *formula* is being used, or maybe just some simplification is happening), then write "none." *At most one rule is being used per line.* The first blank has been filled in for you.

$$\frac{d}{dx}\left[5^{2x^2} + 8\cos\left(4^{x+\cos(x)}\right)\right]$$

$$= \frac{d}{dx}\left[5^{2x^2}\right] + \frac{d}{dx}\left[8\cos\left(4^{x+\cos(x)}\right)\right]$$ sum rule

$$= \frac{d}{dx}\left[5^{2x^2}\right] + 8\frac{d}{dx}\left[\cos\left(4^{x+\cos(x)}\right)\right]$$ constant multiple rule

$$= \ln(5)\cdot 5^{2x^2}\frac{d}{dx}\left[2x^2\right] + 8\frac{d}{dx}\left[\cos\left(4^{x+\cos(x)}\right)\right]$$ chain rule

$$= \ln(5)\cdot 5^{2x^2}\cdot 2\frac{d}{dx}\left[x^2\right] + 8\frac{d}{dx}\left[\cos\left(4^{x+\cos(x)}\right)\right]$$ constant multiple rule

$$= \ln(5)\cdot 5^{2x^2}\cdot 2\frac{d}{dx}\left[x^2\right] + 8\left(-\sin\left(4^{x+\cos(x)}\right)\right)\frac{d}{dx}\left[4^{x+\cos(x)}\right]$$ chain rule

$$= \ln(5)\cdot 5^{2x^2}\cdot 2\cdot 2x$$

$$+ 8\left(-\sin\left(4^{x+\cos(x)}\right)\right)\cdot\ln(4)\cdot 4^{x+\cos(x)}\frac{d}{dx}[x+\cos(x)]$$ chain rule

$$= \ln(5)\cdot 5^{2x^2}\cdot 2\cdot 2x$$

$$+ 8\left(-\sin\left(4^{x+\cos(x)}\right)\right)\cdot\ln(4)\cdot 4^{x+\cos(x)}\left(\frac{d}{dx}[x] + \frac{d}{dx}[\cos(x)]\right)$$ sum rule

$$= \ln(5)\cdot 5^{2x^2}\cdot 2\cdot 2x$$

$$+ 8\left(-\sin\left(4^{x+\cos(x)}\right)\right)\cdot\ln(4)\cdot 4^{x+\cos(x)}\ (1-\sin(x))$$ none

$$= 4x\ln(5)5^{2x^2} - 8\ln(4)4^{x+\cos(x)}\sin\left(4^{x+\cos(x)}\right)(1-\sin(x))$$ none