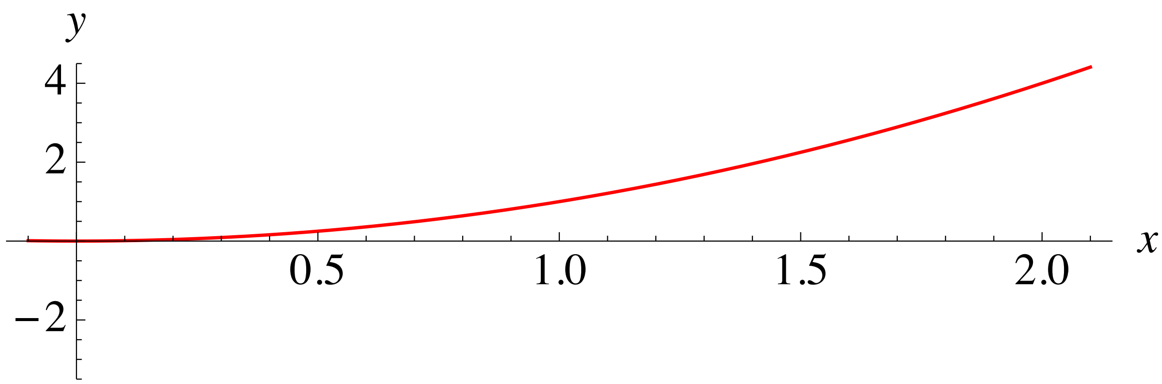
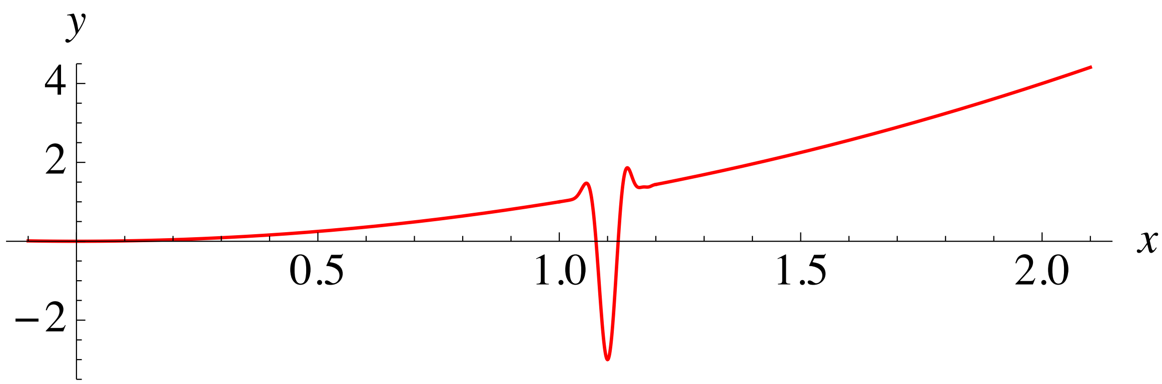
Goal: To explore relationships among functions, their derivatives, rates of change, and slopes of tangent lines

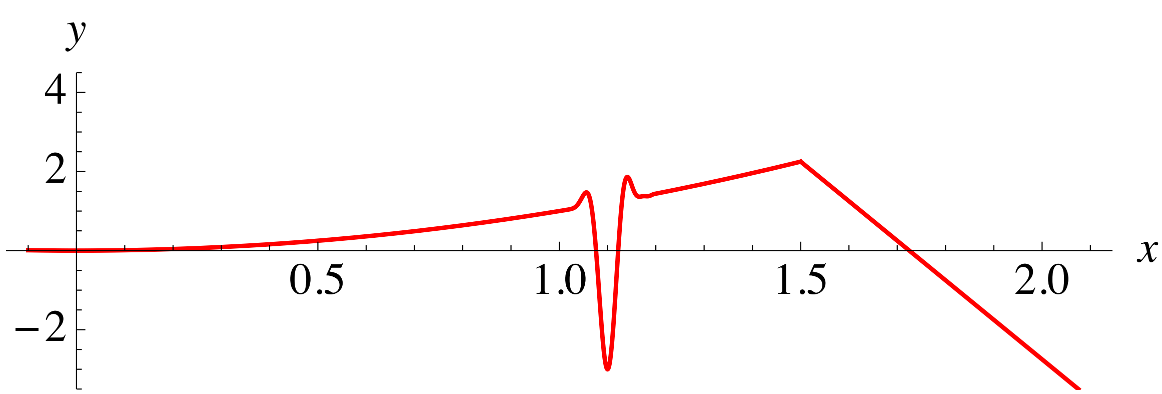
1. (a) Sketch the graph of a function f such that (i) $f'(x)$ exists everywhere on (0,2); (ii) $f(1) = 1$; and (iii) $f'(1) = 2$.



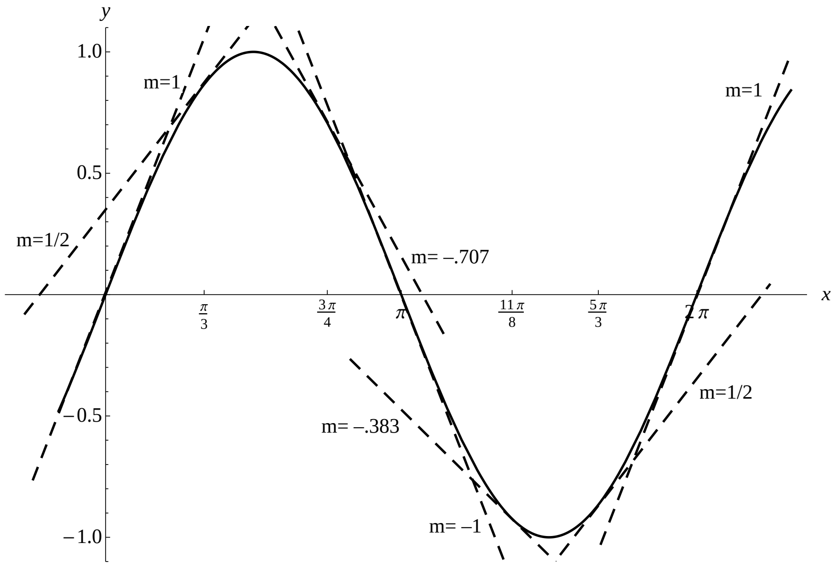
(b) Sketch the graph of a function f such that (i) $f'(x)$ exists everywhere on (0,2); (ii) $f(1) = 1$; (iii) $f'(1) = 2$; and (iv) $f(1.1) = -3$.



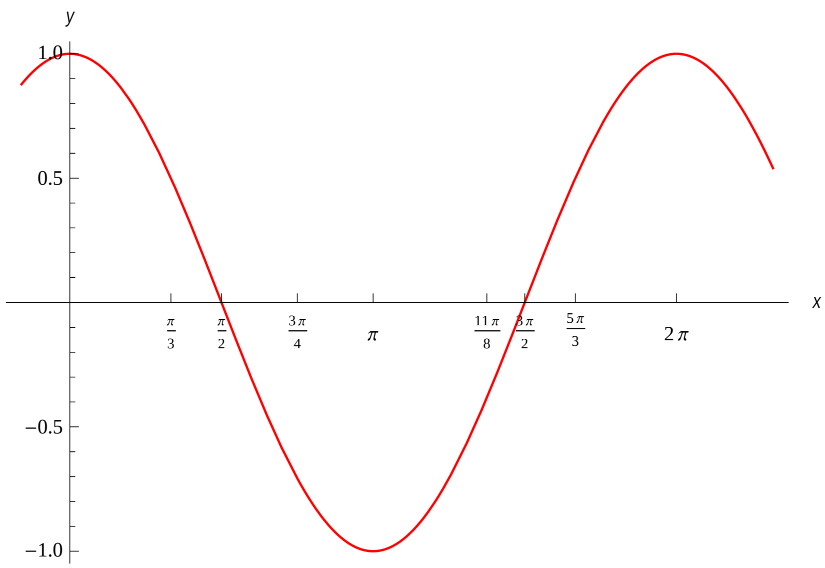
(c) Sketch the graph of a function f such that (i) $f'(x)$ exists everywhere on (0,2), *except* at $x = 1.5$ (but $f(x)$ *is* continuous at $x = 1.5$, meaning it has no jumps or breaks there); (ii) $f(1) = 1$; (iii) $f'(1) = 2$; and (iv) $f(1.1) = -3$.



2. The graph of $f(x) = \sin(x)$ is sketched below, as are the graphs of tangent lines to f at the points $x = 0, \pi/3, 3\pi/4, \pi, 11\pi/8, 5\pi/3$, and 2π. The slope of each tangent line is identified with the notation "m=..." next to that line.



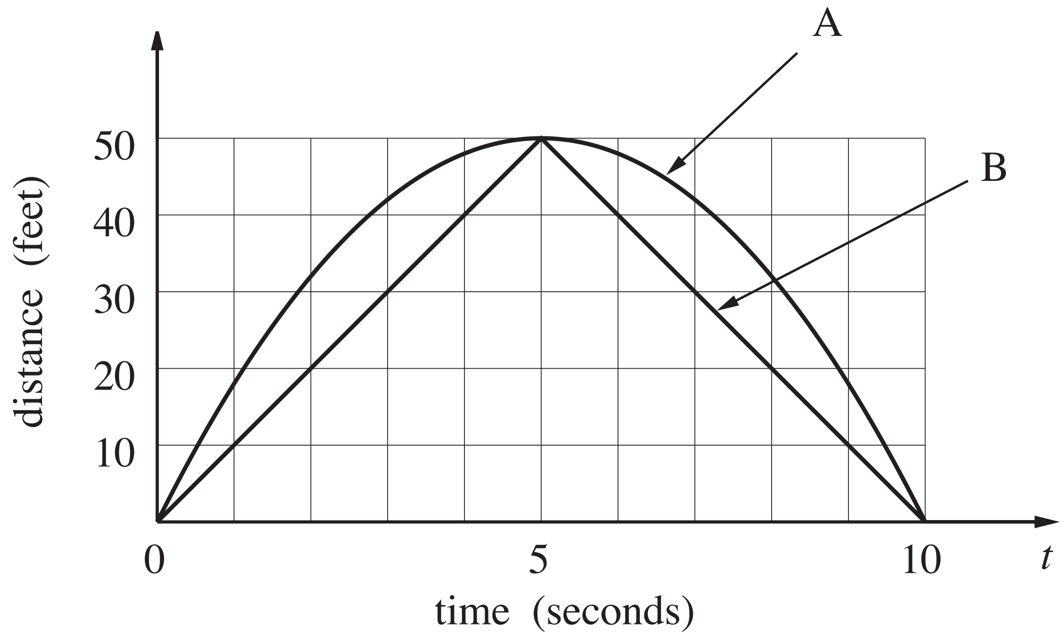
On the axes below, plot each of the above seven x values on the x axis against the slope of the corresponding tangent line on the y axis. Then connect the dots as *smoothly* as you can. (It may help to also observe that, at $x = \pi/2$ and $x = 3\pi/2$, the tangent lines to the above graph are *horizontal*.)



Based on the shape of the graph you got in problem 1 above, conjecture (guess) (pretend you didn't know this already):

If $f(x) = \sin(x)$, *then* $f'(x) =$ ____ $\cos(x)$ ____ .

3. A and B start off at the same time, run to a point 50 feet away, and return, all in 10 seconds. A graph of distance from the starting point as a function of time for each runner appears below. It tells where each runner is during this time interval.



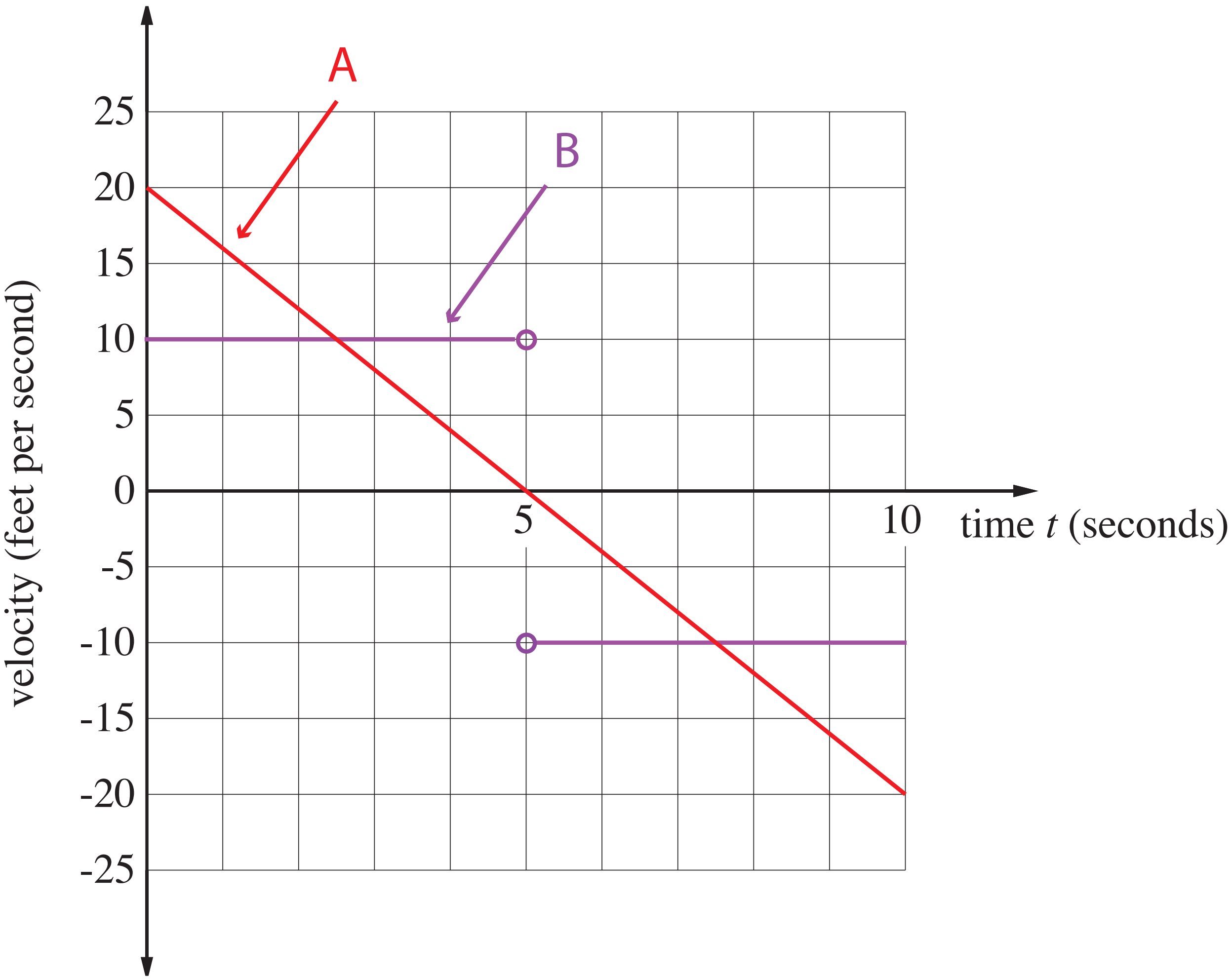
(a) Who is in the lead during the race?

A is in the lead for the first five seconds (because A's distance from the starting point is larger than B's during this period), and B for the last five (for the same reason. If you're running *back* towards the starting point, then you're better off being a shorter distance from that point).

(b) At what time(s) is A farthest ahead of B? At what time(s) is B farthest ahead of A?

A is farthest ahead roughly between seconds 2 and 3. B is farthest ahead roughly between seconds 7 and 8.

(c) Estimate the velocities of A and B during each of the ten seconds. Be sure to assign negative velocities to times when the distance to the starting point is shrinking. Use these estimates to sketch graphs of the velocities of A and B versus time.



(After 5 seconds, the distance between A or B and the starting point is decreasing. Since velocity is rate of change of distance, this means both A and B have negative velocities after 5 seconds.)