Goal: To explore some ways of understanding the parameters a and b in SIR.

1. A town of population 100,000 is hit with a measles epidemic, which evolves according to the usual SIR equations

$$\begin{aligned} S' &= -a\,S\,I, \\ I' &= \quad a\,S\,I - b\,I, \\ R' &= \qquad\qquad b\,I. \end{aligned}$$

This unique strain of the measles is known to last for twelve days.

(a) What is the recovery coefficient b, and what are the units for b? Please explain.

Since the disease lasts twelve days, about 1/12 of those infected recover on any given day, so that $b = 1/12 = 0.083333$. The units of b are day^{-1} (since these are the units that will make the units match up on both sides of the equation $R' = bI$).

On day 15, 14,893 people are susceptible (that is, $S(15)$ =14,893) and 69,613 people are infected (so $I(15)$ =69,613). *One tenth* of a day later, the number of susceptibles is 13,856.

(b) What, at least approximately, is $S'(15)$ (the derivative of S at $t = 15$)? What are the units of $S'(15)$? Hint: approximate this *instantaneous rate of change* by the *approximate rate of change* of S with respect to time t, from day 15 to day 15.1.

By the hint, $S'(15)$ is approximately

$$\frac{S(15.1) - S(15)}{15.1 - 15} = \frac{13856 - 14893}{.1} = -10{,}370$$

individuals per day. (Note that $S'(15)$ is negative, which is indicative of the fact that S is *decreasing*.)

(c) What is the transmission coefficient a? What are the units for a? **HINT:** use the above rate equation for S.To compute, or at least approximate, a, we can observe the following. We have the equation $S' = -aSI$, so in particular, on day 15,

$$(*) \qquad S'(15) = -a\,S(15)I(15).$$

Plugging $S'(15) = -10{,}370$, $S(15) = 14{,}893$, and $I(15) = 69{,}613$ into $(*)$ gives

$$-10{,}370 = -a \cdot 14{,}893 \cdot 69{,}613,$$

or $a = 0.000010$ (to six decimal places). The units of a are $(\text{person}\cdot\text{day})^{-1}$ (since these are the units that will make the units match up on both sides of the equation $S' = -aSI$).

(d) About how large was the susceptible population on day 14?

$$\begin{aligned} S(14) &= S(15) + \Delta S \\ &= S(15) + S'(15)\Delta t \\ &= 14{,}893 + (-10{,}370) \cdot (-1) = 25{,}263 \end{aligned}$$

individuals.