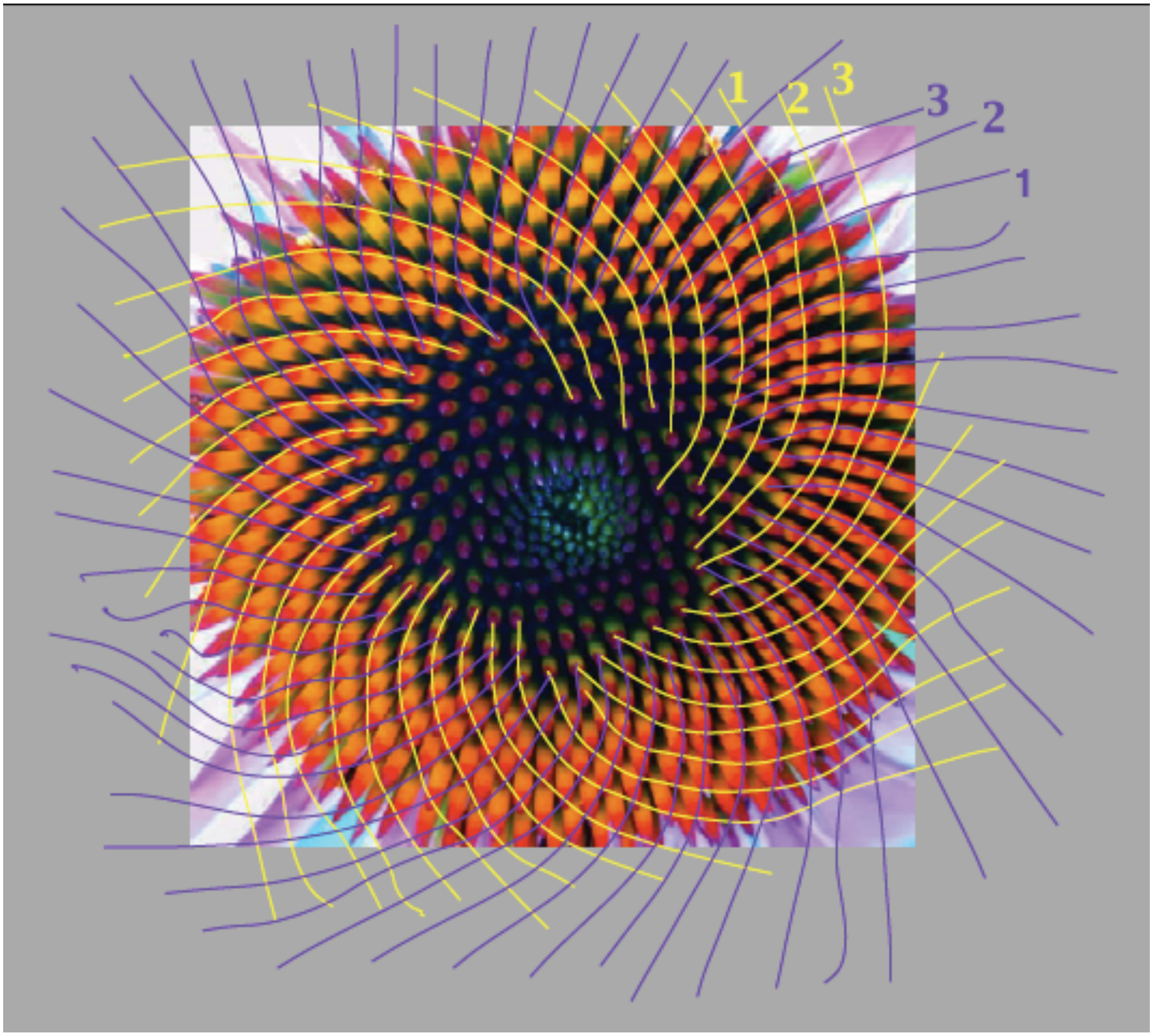
1. Count the number of clockwise (yellow) and counterclockwise (purple) spiral arms in the coneflower below.



Clockwise spirals: __34__ Counterclockwise spirals: __55__

What's the significance of problem 1? To answer, we define the *Fibonacci numbers*, which looks like this:

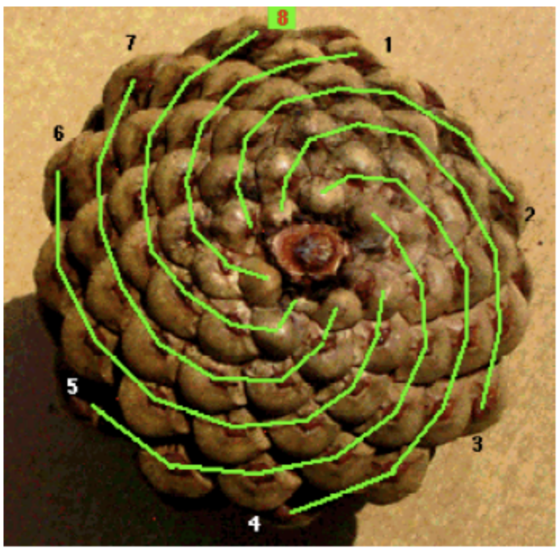
$$0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots.$$

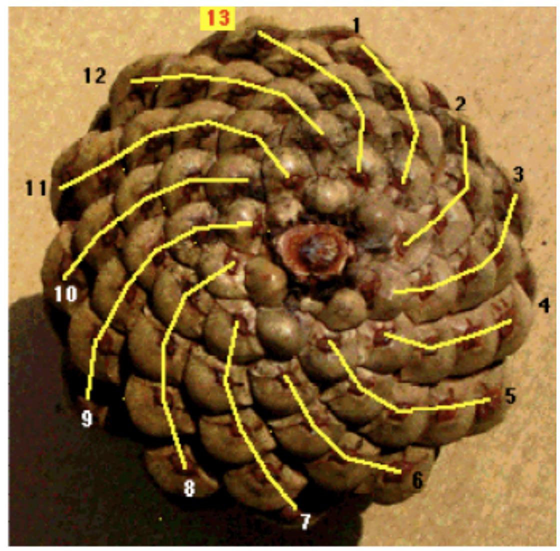
The rule for finding terms in this sequence is: the "zeroth" term is 0; the next term is 1; to get any other term, add together the previous two terms. Or in other words: if we denote the ith Fibonacci number by F_i, then we have

$$F_0 = 1, \quad F_1 = 1, \quad F_i = F_{i-1} + F_{i-2} \quad \text{for } i \geq 2.$$

2. Write down the next nine Fibonacci numbers.

34, 55, 89, 144, 233, 377, 610, 987, 1597,...FACT: Fibonacci numbers are EVERYWHERE. See, for example, Problem 1 above. Similarly, count clockwise and counterclockwise spirals on a pine cone: you'll get consecutive Fibonacci numbers! Really!!





Similar things happen with sunflowers, broccoli florets, etc. See
http://pw1.netcom.com/~merrills/fibphi.html

3. Now we look at *ratios* of successive Fibonacci numbers. That is, we look at the sequence

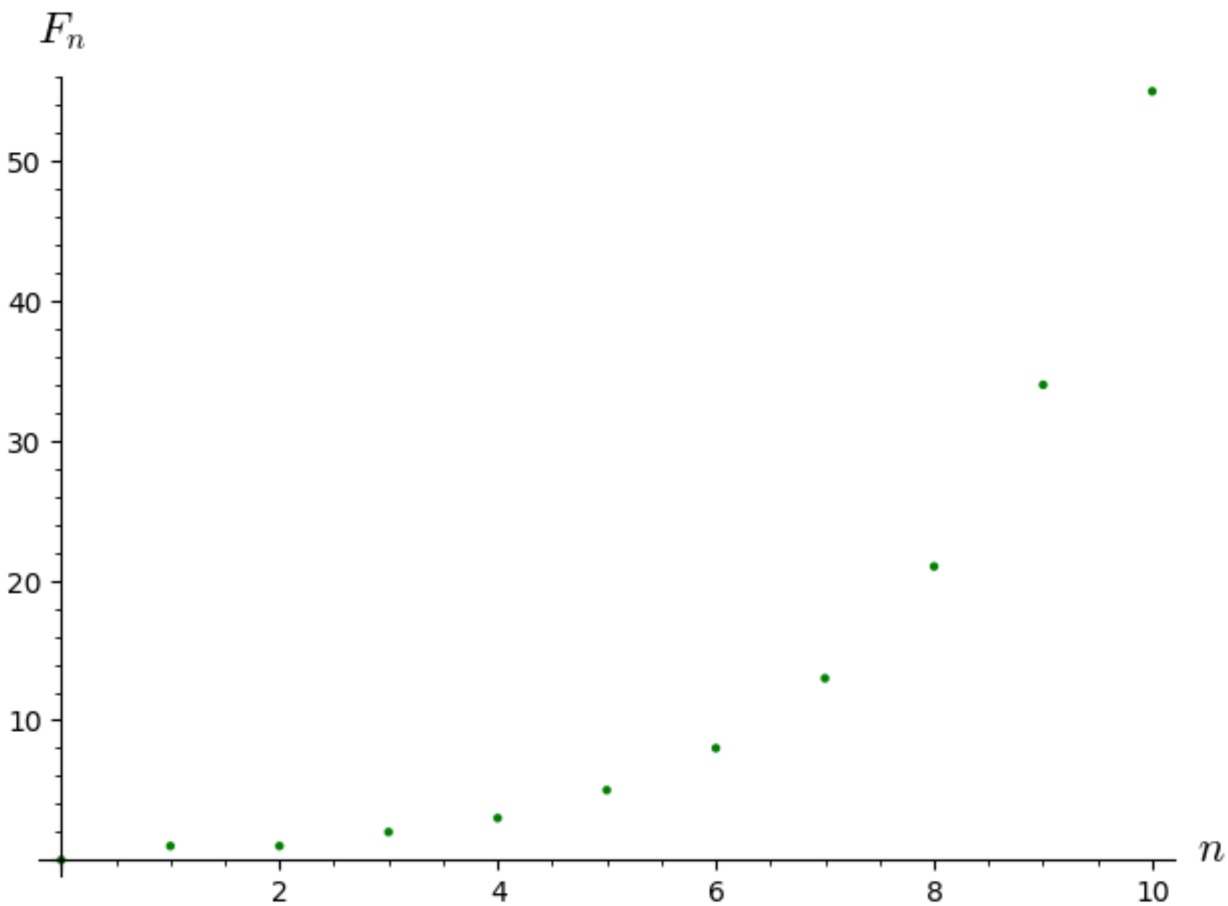
$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \quad \frac{21}{13} = 1.615385, \quad \frac{34}{21} = 1.619048, \quad \frac{55}{34} = 1.617647, \quad \frac{89}{55} = 1.618182,$$

$$\frac{144}{89} = 1.617976, \quad \frac{233}{144} = 1.618056, \quad \frac{377}{233} = 1.618026, \quad \frac{610}{377} = 1.608037, \ldots.$$

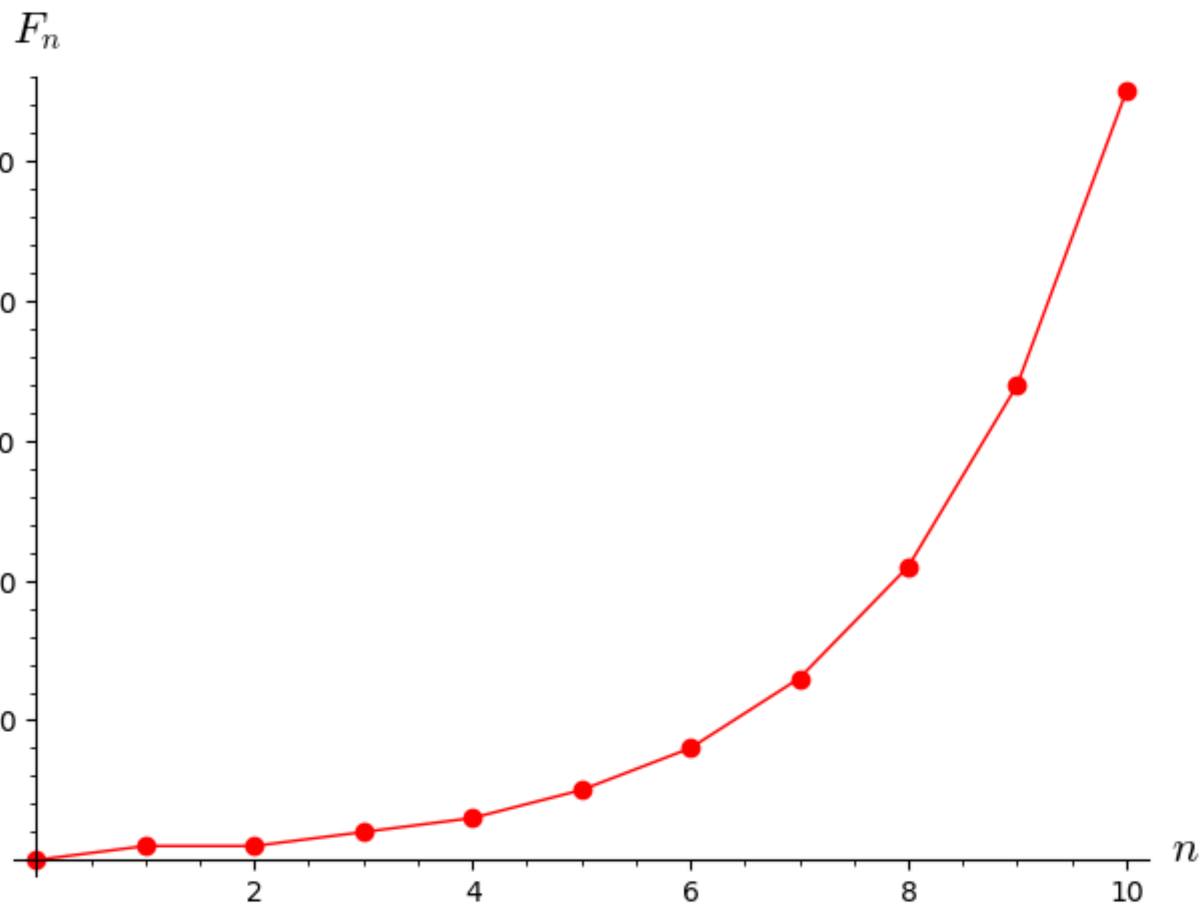
Write down the next seven ratios in this sequence. Then write these seven terms as decimal numbers, with at least six places after the decimal point. Do these ratios appear to be converging? That is, do they appear to be zeroing in on a particular number? If so, what (approximately) does this number appear to be (to as many decimal places as you care to speculate)? They seem to be bouncing up and down around some number that seems to be around 1.60803.

Fibonacci_2020

```
# Program to generate Fibonacci numbers

# First we set up an empty list for the values we're about to compute

FibNumbers=[]

# We're going to construct Fibonacci numbers F_0 through F_n.  For
starters, we'll choose n=10:

n=10

# Create a list of the indices 0 through n:

Domain=[0,1..n]

# We now define the first two Fibonacci numbers F_0 and F_1 to equal
zero to one respectively, and store these numbers as the zeroth and
first entries in the above list:

FibNumbers.append(0)
FibNumbers.append(1)

# Next, we create a loop that:  (a) defines the next Fibonacci number
(starting with F_2) as the sum of the
# previous two, and (b) stores this new Fibonacci number as the next
entry in the list FibNumbers.  The process stops
# after the nth Fibonacci number F_n is generated.

for i in [2,3,4,5,6,7,8,9,10]:
    FibNumbers.append(FibNumbers[i-1]+FibNumbers[i-2])

#Now zip together the domain and the corresponding Fibonacci numbers
(that is, pair each index n with F_n):

points=zip(Domain,FibNumbers)

# Now for the graph

list_plot(points,marker='o',color='green',axes_labels=['$n$','$F_n$'])
```




```
# We can connect the dots

list_plot(points,marker='o',color='red',axes_labels=
['$n$','$F_n$'],plotjoined=True)
```




```
# Let's give a name to the above graph

coolgraph=_
```

```
#Let's save coolgraph to a pdf file

coolgraph.save('fibgraph2.pdf')
```

fibgraph2.pdf