Math 1310: CLS Tutorial: Functions and Graphing with Sage (SOLUTIONS)

This tutorial will involve graphing using Sage.

1. Log in to the Sage server at `sage.colorado.edu` and open up a new blank Sage worksheet (and give it a name, say "Functions").

   One of the things Sage can do is graph functions given a formula. To plot the function $y = x^2$ on the domain $[-5, 5]$, type the following into the empty "cell" (input box):

   ```
   plot(x^2,-5,5)
   ```

   and press "Evaluate" or Shift+Enter. (Note that Sage uses a caret "^" for exponents.)

2. Labelling axes is easy. Try entering and executing this, in a new cell:

   ```
   plot(x^2,-2,2,axes_labels=['x','y'])
   ```

   (If you want, do this by copying the code from your previous exercise into a new cell, and modifying as necessary.) How is your new graph different from the previous one?
   The axes are now labelled.

3. Now suppose that, instead, we wanted to plot the function $s = t^2$. Try entering

   ```
   plot(t^2,-2,2,axes_labels=['t','s'])
   ```

   and executing. What goes wrong?

   Some kind of incomprehensible error message appears.

4. The problem is that, unless you tell Sage otherwise (and you can, but we won't discuss how, at least not yet), it will only plot properly when the independent variable is called x. But that's OK, we can "trick" Sage into plotting $s = t^2$, by calling the independent variable x, but labelling our axes otherwise. Try this:

```
plot(x^2,-2,2,axes_labels=['t','s'])
```

5. In mathematics, it's traditional to *italicize* variable names. To do this in the axes labels on a Sage plot, put dollar signs around these names. Try this:

```
plot(x^2,-2,2,axes_labels=['$t$','$s$'])
```

6. Now suppose that, instead of simply putting a t on the horizontal axis and an s on the vertical axis, we want to write "t (seconds)" on the horizontal axis, and "s (meters)" on the vertical axis. By cutting, pasting, and modifying your code from the previous exercise, enter and execute code that will do exactly this. (Careful: you want to italicize the t and the s, but nothing else.)

   Type in

```
plot(x^2,-2,2,axes_labels=['$t$ (seconds)','$s$ (meters)'])
```

   Note carefully the placement of the dollar signs and the single quote marks.

7. Plotting two or more functions is easy; just put plus signs between the "plot" commands. It's nice to know which function is which, so we'll also add some color. For example, we'll plot the two functions

$$f(x) = x^2 \text{ (in blue)} \qquad \text{and} \qquad g(x) = 3x - 2 \text{ (in red)}$$

   by evaluating the code

```
plot(x^2,-1,3,color='blue',axes_labels=['$x$','$y$'])
+plot(3*x-2,-1,3,color='red')
```

   (put it all on one line; it wouldn't fit here). Note that Sage uses an asterisk "*" for multiplication: you can't just type in `3x`. Also NOTE THAT YOU ONLY HAVE TO LABEL THE AXES IN ONE OF THE TWO "PLOT" COMMANDS!!!!!

8. Where do the above two graphs appear to intersect? How would you verify this algebraically?

   They appear to intersect at $x = 1$ and $x = 2$. To verify this, we set the functions equal and solve:

$$\begin{aligned} x^2 &= 3x - 2 \\ x^2 - 3x + 2 &= 0 \\ (x-1)(x-2) &= 0 \\ x = 1 &\text{ or } x = 2. \end{aligned}$$

9. Sage can plot trigonometric functions (we'll talk more about these later). By default, the domain is always in radians, so in order to see two cycles of the sine function, you could type

```
plot(sin(x),-2*pi,2*pi)
```

   Go ahead and try this. OK, who am I to argue?

10. Try plotting three or more functions at a time, all on the same domain, and each in a different color. (You can cut and paste from the entries above, but make sure you make the necessary adjustments to get the same domain but different colors. What happens if you DON'T specify the same domain for all three functions?)

    Try

```
plot(x^2,-1,3,color='blue',axes_labels=['$x$','$y$'])
+plot(3*x-2,-1,3,color='red')
+plot(sin(x),-1,3,color='green')
```

    Note that, if we had written

```
+plot(sin(x),-2*pi,2*pi,color='green')
```

    at the end instead, then the sine curve would have stretched out all the way from -2π to 2π, but the other two curves would only have appeared from -1 to 3.

Now please press the "Save & quit" button at the top of your worksheet. Your worksheet will be saved in your directory as Functions.sws. You can now log out of Sage. You're done! See ya!