

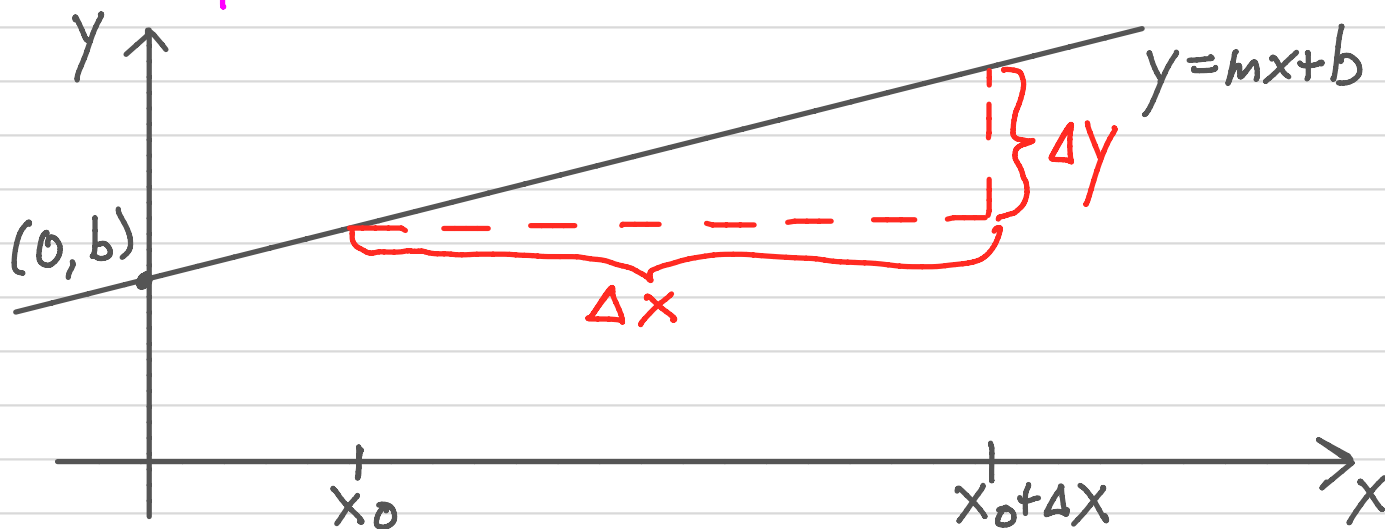
Week 2 - Friday, 1/25

Linear functions.

A) Basics.

A linear function (or line) is one of the form

$$y = mx + b$$

the slope,
or multiplier.the y-interceptCOOL FACT about lines and rates of change:Suppose $y = f(x) = mx + b$ is a linear function. If x changes by Δx , then by how much does y change?Well, say x goes from x_0 to $x_0 + \Delta x$. Then the change in y is:

$$\begin{aligned}
 \Delta y &= \text{new } y \text{ minus old } y \\
 &= f(x_0 + \Delta x) - f(x_0) \\
 &= m(x_0 + \Delta x) + b - (mx_0 + b) \\
 &= \cancel{mx_0} + m\Delta x + \cancel{b} - \cancel{mx_0} - \cancel{b} \\
 &= m\Delta x.
 \end{aligned}$$

CONCLUSION. For a linear function $y = mx + b$, Δy is always proportional to Δx :

$$\Delta y = m \Delta x, \text{ or } m = \frac{\Delta y}{\Delta x}.$$

* Linear functions have a constant rate of change $\Delta y / \Delta x$, which equals the slope m . This is a special property of linear functions, as we'll see.

Example. Let C denote temperature in $^{\circ}\text{C}$, and F temperature in $^{\circ}\text{F}$.

- (a) What's the multiplier m in the equation $\Delta F = m \Delta C$?
- (b) What's the rate of change of F with respect to C ?
- (c) If C decreases by 2°C , by how much does F change?
- (d) What's the rate of change of C with respect to F ?

Solution.

- (a) We have $F = \frac{9}{5}C + 32$. The slope m is $9/5$, so by the CONCLUSION above,

$$\Delta F = \frac{9}{5} \Delta C. \text{ So the multiplier is } 9/5.$$

(b) $m = 9/5$ ($^{\circ}\text{F}$ per $^{\circ}\text{C}$)

(c) $\Delta F = \frac{9}{5} \Delta C = \frac{9}{5} \cdot (-2) = -18/5.$

F decreases by $18/5 = 3.6^{\circ}\text{F}.$

- (d) We solve for C :

$$C = \frac{5}{9}(F - 32) = \frac{5}{9}F - \frac{5 \cdot 32}{9} = \frac{5}{9}F - \frac{160}{9}.$$

So the rate of change of C with respect to F is $5/9$ ($^{\circ}\text{C}$ per $^{\circ}\text{F}$).

B) Equations for lines.

(1) As above:

$$y = mx + b \quad \text{slope-intercept form}$$

Example.

A line through $(0, 2)$, and such that y changes by -3 for each unit increase in x , has equation

$$y = -3x + 2.$$

(2) Say you're given the slope m of a line, and a point (x_0, y_0) on it. Then, for any other point (x, y) on the line,

$$m = \frac{\Delta y}{\Delta x} = \frac{y - y_0}{x - x_0} \quad \text{or, solving for } y,$$

$$y = m(x - x_0) + y_0 \quad \text{point-slope, or initial value, form.}$$

Example. The line through $(-2, 1)$, with slope 4, has equation

$$\begin{aligned} y &= 4(x - (-2)) + 1 \\ &= 4x + 8 + 1 = 4x + 9. \end{aligned}$$

(3) "Two points determine a line."

If a line passes through (x_1, y_1) and (x_2, y_2) , then by the point-slope form,

$$y = m(x - x_1) + y_1 \quad \text{where } m = \frac{y_2 - y_1}{x_2 - x_1}. \quad \text{two-point, or interpolation, form}$$

Example.

The line through $(3, 5)$ and $(1, 1)$ has slope

$$m = \frac{1-5}{1-3} = \frac{-4}{-2} = 2,$$

and equation

$$\begin{aligned} y &= 2(x-3)+5 \\ &= 2x-6+5 = 2x-1. \end{aligned}$$