The basic reproduction number r_0.

A) Definition and formula.

The basic reproduction number r_0 is the total number of new infections caused by each infected individual, at the outset of the epidemic.

Computation of r_0:

(a) Since $S' = -aSI$ there are, over the course of day 0, $aS(0)I(0)$ new infections. So:

(b) Per infected individual, there are

$$\frac{aS(0)I(0)}{I(0)} = aS(0)$$

new infections over the course of day 0. But one stays infected for k days, so:

(c) At the outset, there are $kaS(0)$ total new infections per infected individual.

Conclusion:

$$\boxed{r_0 = kaS(0)}$$

formula for the basic reproduction number

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(Here a = transmission coefficient, and $k = 1/b$, where b = recovery coefficient.)

<u>Example 1</u> In our epidemic with $S(0) = 500$, $a = 0.001$, $b = 0.2$, we have

$$r_0 = kaS(0) = \frac{1}{b} \cdot a \cdot S(0) = \frac{1}{0.2} \cdot 0.001 \cdot 500$$
$$= 2.5 \text{ individuals.}$$

(B) Important fact about r_0:

<u>Proposition.</u> If $r_0 > 1$, then I will initially grow.
If $r_0 < 1$, then I will initially shrink.

Proof: see text, p. 22.

(C) r_0 and herd immunity.

Recall from last time: herd immunity can be achieved by immunizing a fraction

$$f > 1 - \frac{b}{aS(0)} \qquad (*)$$

of the susceptible population. But $b = 1/k$, so $(*)$ can be written

$$f > 1 - \frac{1}{kaS(0)}$$ or, since $kaS(0) = r_0$,

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$$\boxed{f > 1 - \frac{1}{r_0}}$$ immunization proportion for herd immunity

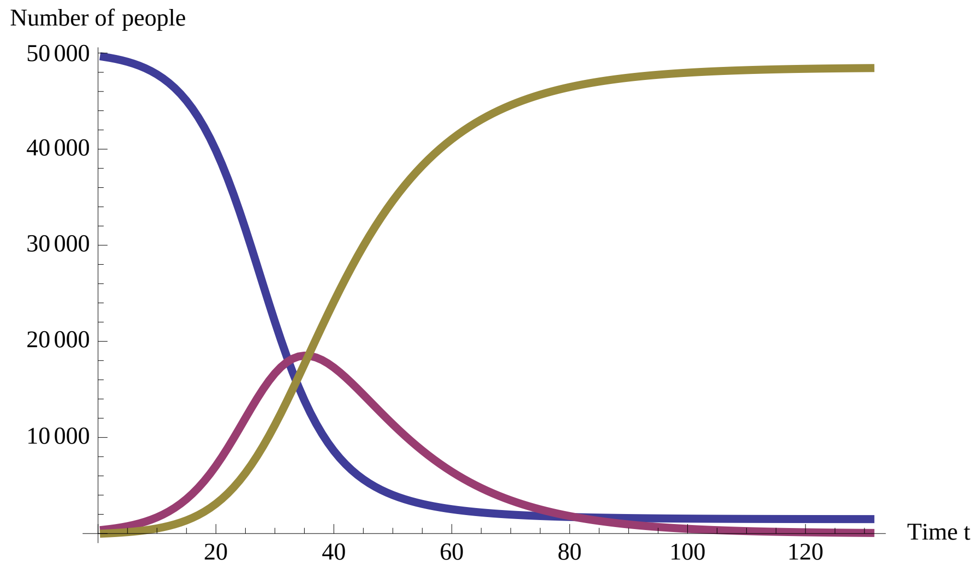
Example 2. For our epidemic of Example 1, we attain herd immunity by immunizing better than

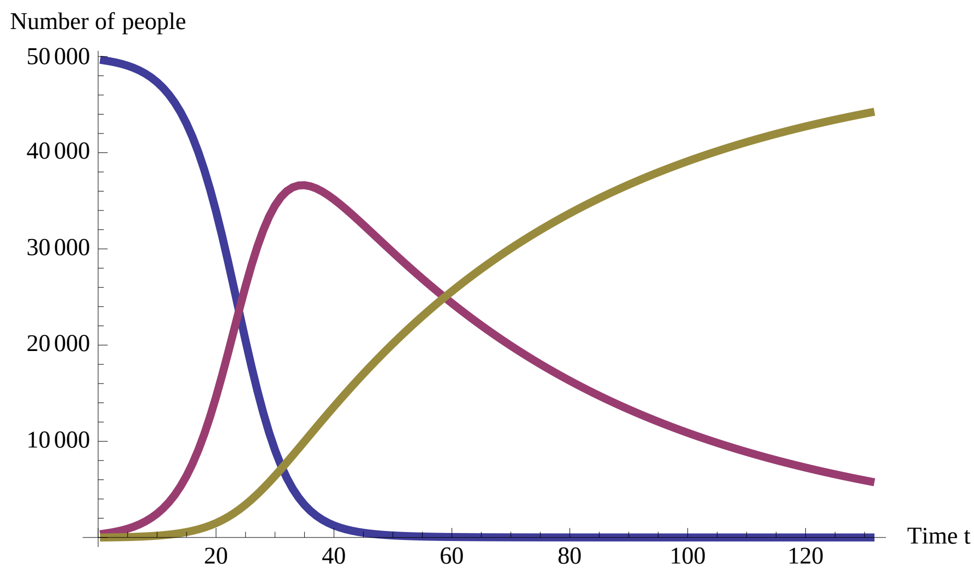
$$1 - \frac{1}{2.5} = 0.6 = 60\%$$

of the initial susceptible population. (Same result as in the notes for 9/1.)

Goal: To explore some more ideas about modeling with rate equations and SIR.

1. Pictured below are two graphs depicting evolution of diseases that progress according to the usual SIR model. For both graphs, the initial values $S(0)$, $I(0)$ and $R(0)$, and the transmission coefficient a, are the same. But the two graphs correspond to different recovery coefficients b.





(a) On each of the graphs, label which curve is S, which is I, and which is R.

In each graph, the "backwards S" curve is S, the "bell" curve is I, and the "S" curve is R.

(b) Which of the above two graphs corresponds to the *larger* value of b? Please explain.

The top one. Remember $b = 1/k$, where k is the number of days to recovery. So larger b means smaller k, which means faster recovery, which we see in the top graph.

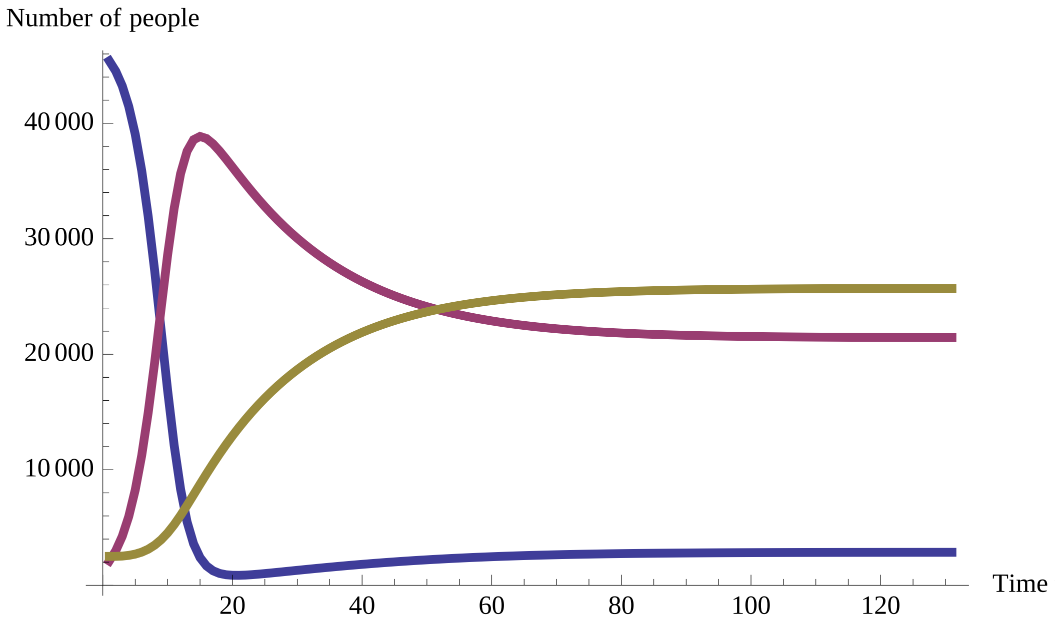
(c) Which of the above two epidemics has the larger basic reproduction number r_0? Please explain. The bottom one. Again, $b = 1/k$, where k is the number of days to recovery. The bottom graph has the smaller b, as we just noted above. Smaller b means larger k, which means larger $r_0 = kaS(0)$.

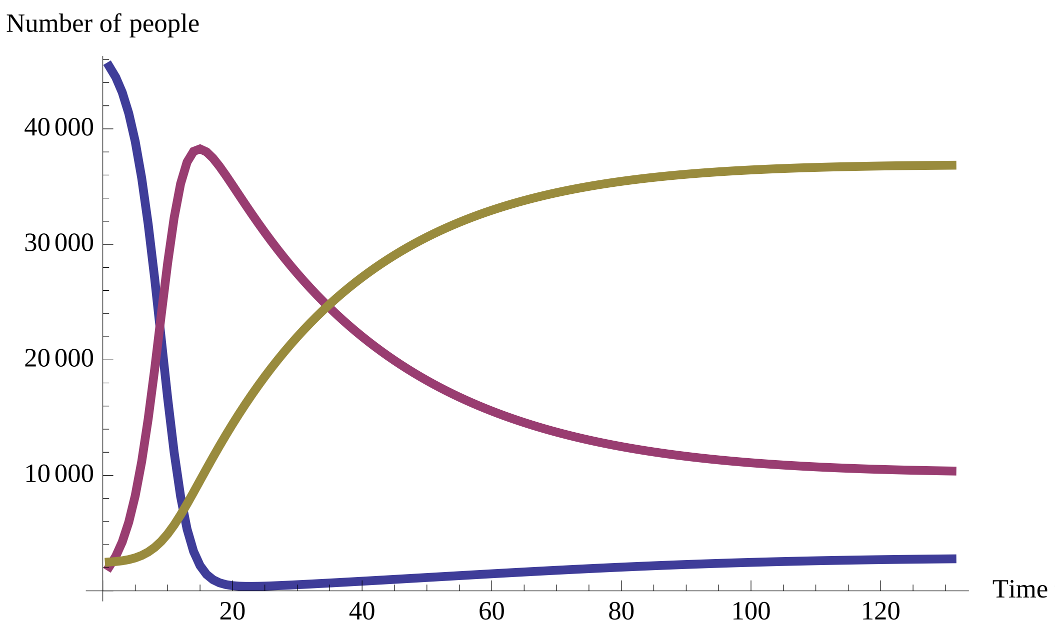
2. Consider an epidemic that progresses according to the usual SIR model, *except* that, now, recovered people become susceptible again (and can infect again) after m days.

(a) *Modify* the usual SIR equations to reflect this new feature (wherein recovered can become susceptible again). HINTS: (a) Your new equations will look a *lot* like the old ones, but with some *new terms* added on. These terms should account for the facts that, now, on average, $1/m$ of the recovered population gets *added to* susceptible population, and *subtracted from* the recovered population, on any given day. (b) Your new equations should involve unspecified parameters a, b, and c, where a and b are as above, and $c = 1/m$.

$$\begin{aligned} S' &= -a\,S\,I \qquad\quad + cR, \\ I' &= \quad a\,S\,I - b\,I, \\ R' &= \qquad\qquad\; b\,I - cR. \end{aligned}$$

(b) In the two graphs on the next page, the transmission and recovery coefficients a and b are the same, but the number of days m that it takes to become susceptible again differs from one graph to the next. For which of the two graphs – the one on the top or the one on the bottom — does it take *longer* to become susceptible again? Please explain.





The bottom graph. If it takes longer to become susceptible again, then we would expect the number of infected to level off at a relatively low level, and the number of recovered to level off at a relatively high level, as is happening in the bottom graph.