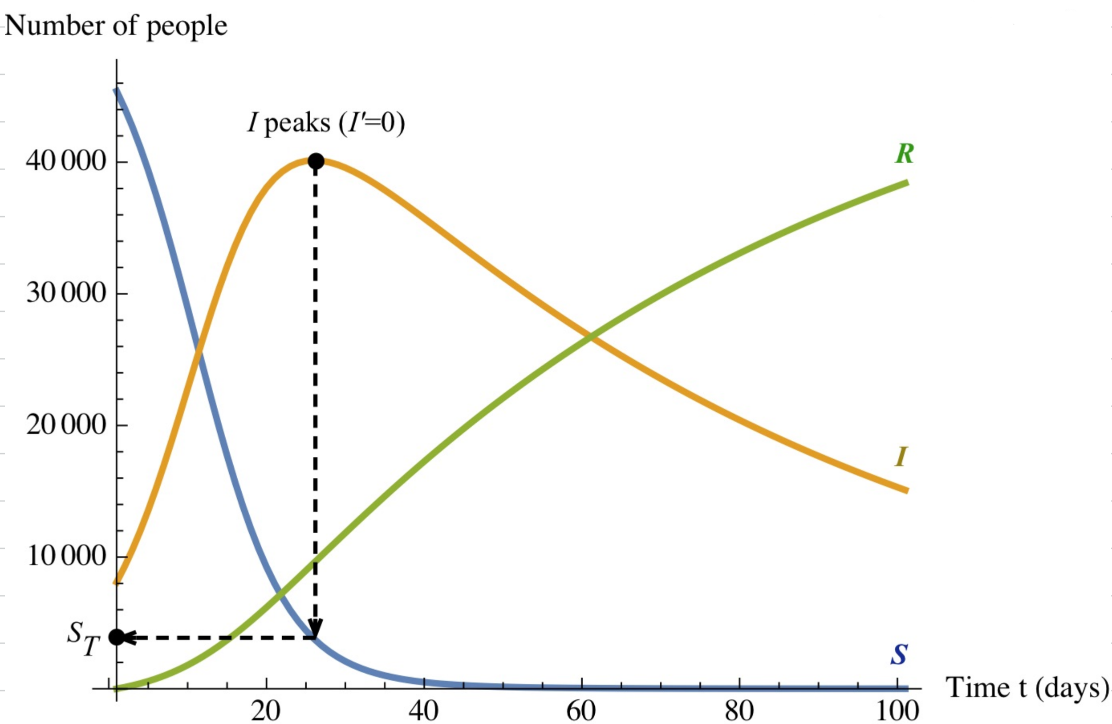
Week 2 - Tuesday, 9/1

Some consequences of SIR.

I) Threshold value of S.

Suppose I is initially increasing. At some point, S will become so small that it no longer sustains growth in I. At this point, I peaks. The value of S at which this happens is called the threshold value, denoted S_T.

Here's a picture:



We can <u>compute</u> S_T as follows:

"I peaks" means "I changes from increasing to decreasing," which means "I' changes from positive to negative." At such a point, I' must be <u>zero</u>.

In sum:

$$\boxed{S_T \text{ is the value of } S \text{ at which } I' = 0.} \qquad (*)$$

Now recall that

$$I' = aSI - bI.$$

So by $(*)$,

$$aS_T I - bI = 0.$$

Factor:

$$I(aS_T - b) = 0.$$

Divide by I:

$$aS_T - b = 0.$$

Solve for S_T:

$$\boxed{S_T = b/a}$$ formula for threshold value

<u>Example.</u> Last time, we considered an epidemic where $a = 0.001$, $b = 0.2$. So for that

epidemic, $S_T = b/a = \frac{0.2}{0.001} = 200$ individuals.

Remark: by similar arguments, we see that:
(a) if S is <u>larger</u> than S_T, then I is <u>growing</u>;
(b) if S is <u>less</u> than S_T, then I is <u>shrinking</u>.

II) Herd immunity.

Suppose we have an SIR-like epidemic, with initial # of susceptibles $= S(0)$.

Question: what fraction f of the susceptible population would we need to immunize to assure <u>herd immunity</u>, meaning I will decrease from the outset?

Answer: if we immunize a fraction f of $S(0)$, then we're left with

$$S(0) - f\,S(0)$$

susceptibles. By Remark (b) above, we want

$$S(0) - f\,S(0) < S_T\text{; that is,}$$
$$S(0) - f\,S(0) < b/a.$$

Solve for f (for details, see text, §1.3):

$$\boxed{f > 1 - \frac{b}{a\,S(0)}}$$

Immunization proportion for herd immunity

E.g. for our above example, with $S(0) = 500$, we'd need to immunize better than

$$1 - \frac{0.2}{0.001 \cdot 500} = 0.6 = 60\%$$

of the susceptible population.