Week 2 - Thursday, 9/3

Functions and graphs.

A) Basics.

A function is a rule that assigns a unique output to every input.

Call the rule f, the input x, and the corresponding output y; then we can write

function

$$y = f(x).$$ "y equals f of x."

dependent variable   independent variable

Examples.

1) $y = x^2$ (The implicit rule f is given by $f(x) = x^2$.)
2) $y = 3x - 5$: a linear function
3) $y = \dfrac{1}{\sqrt{4-7x}}$
4) $y = x^{1/3} \left(= \sqrt[3]{x}\right)$, $y = x^{-10} \left(= \dfrac{1}{x^{10}}\right)$,
   $y = x^{4/7} \left(= \sqrt[7]{x^4} = \left(\sqrt[7]{x}\right)^4\right)$, $y = x^{-4/7} \left(= \dfrac{1}{x^{4/7}}\right)$, etc.
5) $F = \frac{9}{5}C + 32$ (Celsius to Fahrenheit)
6) $P(t) = \dfrac{100}{1 + 9e^{-t/10}}$
   (population, in thousands, after t years, under a certain "logistic growth" model: more on this later.
7) $D = g(C)$:
   density of water, in kg/m^3, as a function of temperature, in degrees Celsius.
8) $y = \sin(t)$: a trigonometric function, whose graph looks like this:

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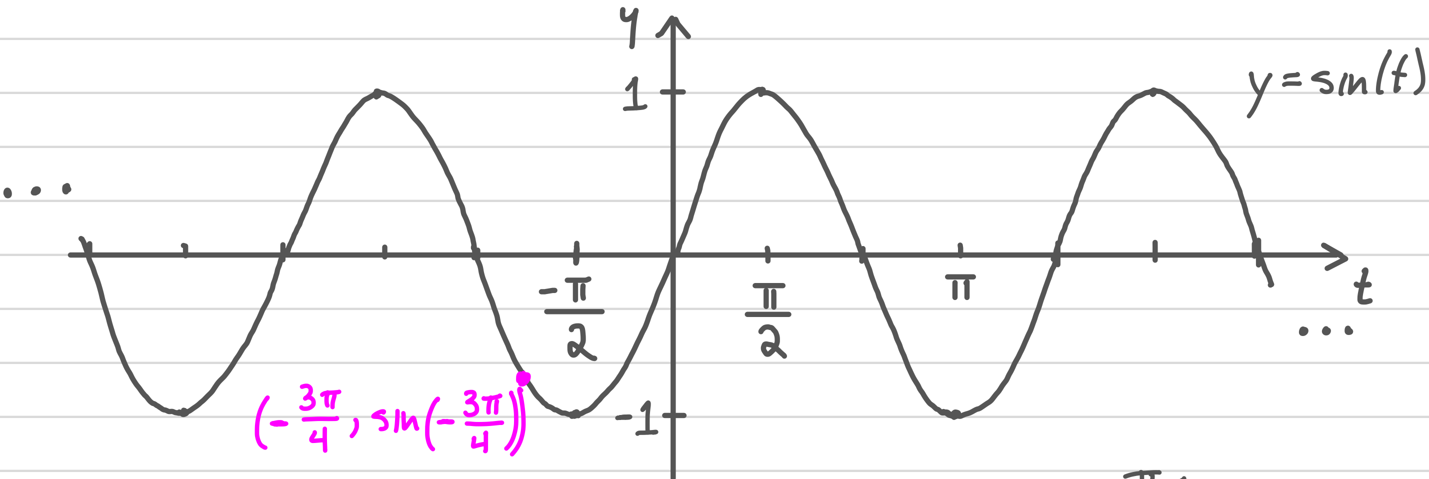


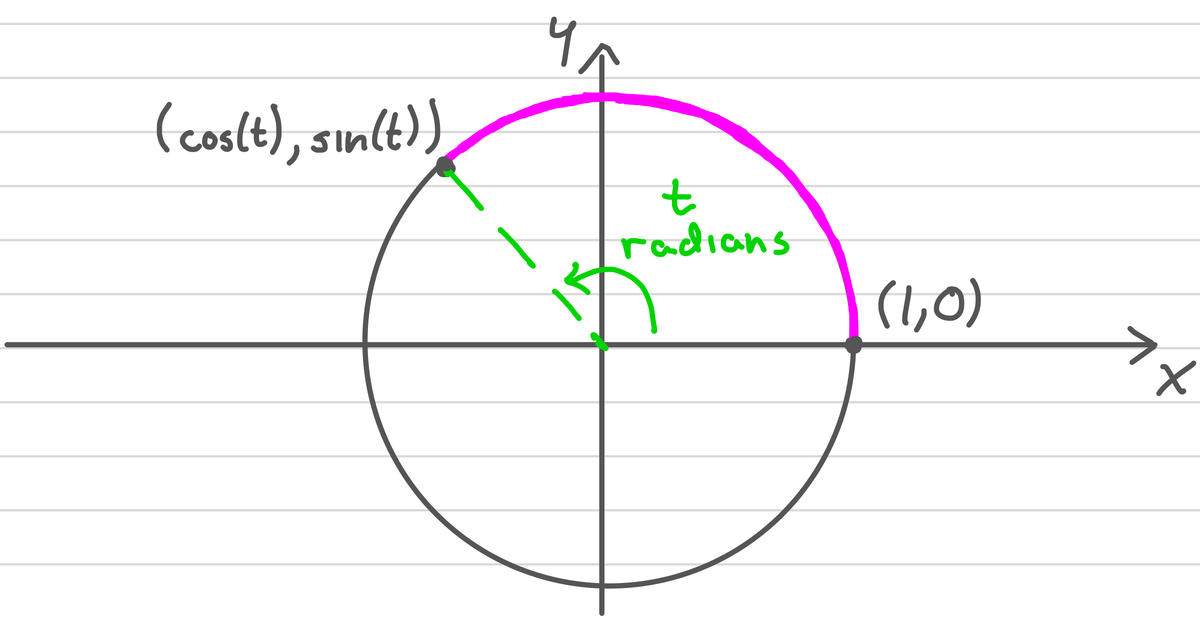
The units for t are radians, where 1 degree $= \pi/180$ radians. E.g.

$$30^\circ = 30 \cdot \frac{\pi}{180} = \frac{\pi}{6} \text{ radians}, \quad -\frac{5\pi}{4} \text{ radians} = -\frac{5\pi}{4} \cdot \frac{180}{\pi} = -225^\circ,$$

etc.

The graph of $y = \cos(t)$ looks the same, but shifted left by $\pi/2$ units.

The functions $y = \cos(t)$ and $y = \sin(t)$ are called circular functions: they arise as coordinates of a point moving around the circle $x^2 + y^2 = 1$, like this:



B) Remark. In general, if $y=f(x)$, we say "y is a function of x."
As noted above, this means x determines y <u>uniquely</u> (that is, unambiguously).
For example, take $y=x^2$. For any x, only one y is possible: namely, $y=x^2$. E.g. $x=3$ gives $y=3^2=9$, unambiguously.
But note that <u>starting</u> with $y=9$ does <u>not</u> give x unambiguously: $x^2=9$ could mean $x=3$ <u>or</u> $x=-3$.

Moral: "y is a function of x" <u>need</u> <u>not</u> imply "x is a function of y."
[Still, sometimes it <u>does</u> imply this: e.g. $F=\frac{9}{5}C+32$ can be solved to give
$$C=\frac{5}{9}(F-32):$$ a unique C for each F.]

C) Chaining (composing) functions.

The <u>chain</u>, or <u>composition</u>, of two functions f and g is when the <u>output</u> $g(x)$ from g is <u>input</u> into f.

$$x \longrightarrow \boxed{g} \longrightarrow g(x) \longrightarrow \boxed{f} \longrightarrow f(g(x))$$

The chain of f and g, denoted $f(g(x))$

<u>Example.</u>
Let $f(x)=3x+4$, $g(x)=x^2$, $h(x)=\frac{1}{x}$, $j(x)=\cos(x)$.
Then:
$$f(g(x))=f(x^2)=3x^2+4$$
$$g(f(x))=g(3x+4)=(3x+4)^2=9x^2+24x+16$$
(Note: here, and in general, $f(g(x))\neq g(f(x))$.)

$$g(g(x))=g(x^2)=(x^2)^2=x^4$$

$j(g(x)) = j(x^2) = \cos(x^2)$

$g(j(x)) = g(\cos(x)) = (\cos(x))^2$, $\longrightarrow$ often denoted $\cos^2(x)$; <u>not</u> the same as $\cos(x^2)$

$h(f(2)) = h(3(2)+4) = h(10) = 1/10$,

$j(h(f(x))) = j(h(3x+4))$
$= j\left(\frac{1}{3x+4}\right) = \cos\left(\frac{1}{3x+4}\right)$,

etc.