Week 1 - Tuesday, 1/15

Modeling a disease using SIR.

I) Initial set-up.

S: # of susceptibles
I: # of infected
R: # of recovered

S': rate of change of S
I': rate of change of I
R': rate of change of R

These are all variables: they vary with time t.

Assumptions:

- Everyone infected recovers eventually.
- The duration of infection is the same for everyone.
- Once recovered, you're immune and can't infect.
- Only a fraction of contacts with disease cause infection.
- The units are:
    - days for time t;
    - individuals for S, I, and R;
    - individuals/day for S', I', and R'.

II) Thinking about the rates of change S', I', and R'.

(a) R'.

Say the disease lasts k days. Then each day, on average, the number recovered will increase by $1/k$ times the size of the infected population. So

$$\boxed{R' = \frac{1}{k} I = bI} \qquad \text{where } b = 1/k.$$

Note: b is constant (it doesn't change with t). We say R' is proportional to I.

(b) S'.

Suppose:

(i) Each susceptible has contact with a fraction, call it p, of the infected population on a given day. Since the number of possible S-to-I contacts on a given day is $S \cdot I$, this means the number of actual S-to-I contacts on a given day is $p \cdot S \cdot I$.

(ii) A fraction, call it q, of such contacts yield infection.

Together, (i) and (ii) mean $q \cdot p \cdot S \cdot I$ new infections each day, meaning S decreases by $q \cdot p \cdot S \cdot I$ each day. SO:

$$\boxed{S' = -qpSI = -aSI} \quad \text{where } a = qp.$$

(The minus sign reflects the decrease in S.)

(c) Assuming the total population $S+I+R$ stays constant, the changes in S, I, and R must cancel, meaning $S'+I'+R'=0$, so

$$I' = -S' - R'$$

or, by (a) and (b) above,

$$\boxed{I' = aSI - bI}$$

III) SUMMARY

Under the assumptions described above, we have

$$\boxed{\begin{aligned} S' &= -aSI \\ I' &= aSI - bI \\ R' &= bI \end{aligned}}$$

SIR equations

Here:

- b (>0) is the recovery coefficient. Units: $1/\text{day}$, or day^{-1}.
- a (>0) is the transmission coefficient. Units: $1/(\text{person}\cdot\text{day})$, or $(\text{person}\cdot\text{day})^{-1}$.

a and b are called parameters.

Question: so what?

Answer: prediction. More on this soon.