Week 1 - Friday, 1/18

Stepsize: What's up with that?

Question: how does changing the "stepsize" Δt change (a) computations and (b) results?

Example.

Given:

- The usual SIR equations

$$\left.\begin{aligned} S' &= -aSI \\ I' &= aSI - bI \\ R' &= bI, \end{aligned}\right] \quad (SIR)$$

- Initial conditions

$$S(0) = 500, \quad I(0) = 10, \quad R(0) = 0,$$

- Parameters

$$a = 0.001, \quad b = 0.2,$$

find $S(4)$, $I(4)$, $R(4)$ using:

(A) stepsize $\Delta t = 4$,

(B) stepsize $\Delta t = 2$.

Solution.

(A)
$$\begin{aligned} S(4) &= S(0) + \Delta S \\ &= S(0) + S'(0)\Delta t \\ &= S(0) + (-a \cdot S(0) \cdot I(0)) \cdot \Delta t \\ &= 500 + (-0.001 \cdot 500 \cdot 10) \cdot 4 \\ &= 500 - 20 = 480. \end{aligned}$$

Also
$$R(4) = R(0) + \Delta R$$
$$= R(0) + R'(0)\cdot \Delta t$$
$$= R(0) + (b\cdot I(0))\cdot \Delta t$$
$$= 0 + (0.2\cdot 10)\cdot 4 = 8.$$

Then
$$I(4) = S(0) + I(0) + R(0) - S(4) - R(4)$$
$$= 500 + 10 + 0 - 480 - 8 = 22.$$

In sum,
$$\boxed{S(4) = 480,\ I(4) = 22,\ R(4) = 8 \text{ (individuals).}}$$

(B) We did this in class; we got
$$\boxed{S(4) = 474.32,\ I(4) = 25.28,\ R(4) = 10.4 \text{ (individuals).}}$$

<u>Notes.</u>

(a) All results are <u>approximate.</u> Why? Because, in equations like
$$S(4) = S(2) + \Delta S = S(2) + S'(2)\Delta t,$$
the second "=" should really be "$\approx$." This is because S' itself typically changes with t, so ΔS is only <u>roughly</u> equal to $S'(t)\Delta t$ for a specific time t.

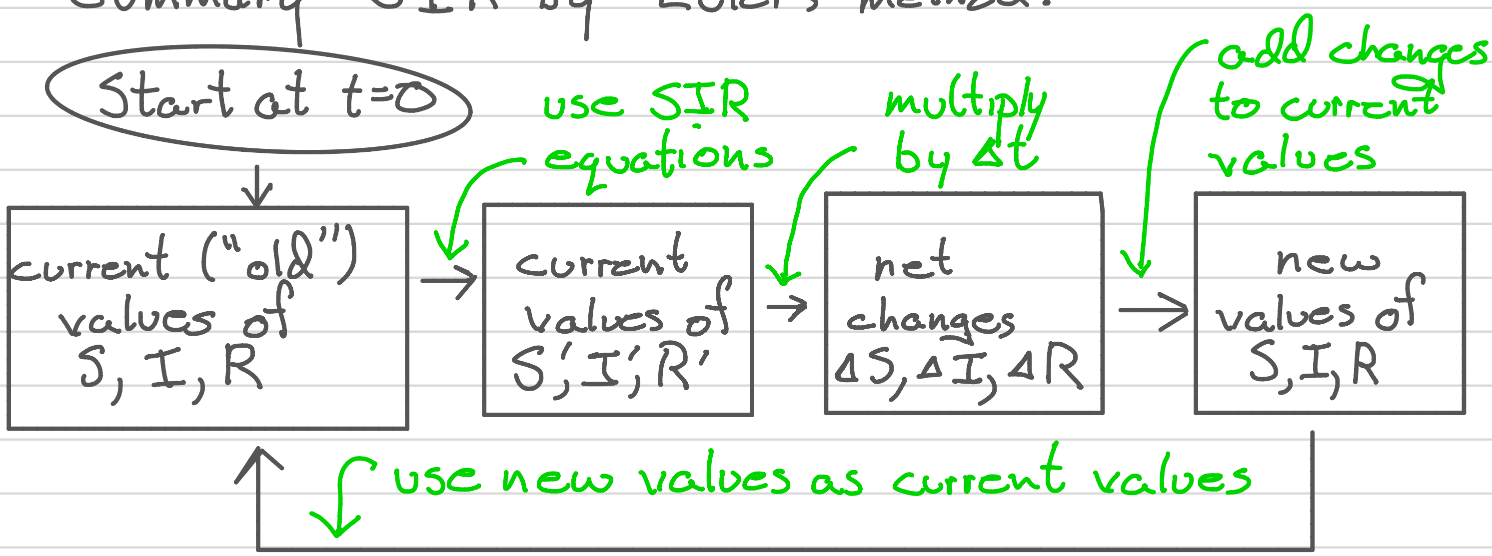
(b) <u>Smaller</u> Δt means more <u>frequent</u> recalibration of $S'(t)$, which typically means BETTER APPROXIMATIONS.

For example: we could predict $S(4), I(4), R(4)$ using $\Delta t = 0.01$ (and a computer); after 401 iterations ($t = 0, 0.01, 0.02, 0.03, \ldots, 3.98, 3.99, 4$), we get

$$\boxed{S(4) = 463.57,\quad I(4) = 31.30,\quad R(4) = 15.13 \text{ individuals}}$$

(a <u>better</u> approximation)

Summary: SIR by "Euler's method."



(Repeat until desired t-value is reached.)