Week 15 - Wednesday, 4/24

Separation of variables, concluded: <u>supergrowth</u>.

Some models suggest world population P grows at a rate proportional to <u>more</u> than just P itself.

According to one such model,

$$\frac{dP}{dt} = kP^{1.2}, \qquad P(0) = P_0. \qquad (SG)$$

(Note that $P^{1.2} > P$, so here, P grows <u>faster</u> than exponentially).

Solve the above IVP for P.

<u>Solution.</u>

Separate:

$$\frac{dP}{P^{1.2}} = k\,dt$$

Integrate:

$$\int \frac{dP}{P^{1.2}} = \int k\,dt$$

$$\int P^{-1.2}\,dP = \int k\,dt$$

$$\frac{P^{-0.2}}{-0.2} = kt + C.$$

Evaluate:

$$\frac{P_0^{-0.2}}{-0.2} = k \cdot 0 + C = C.$$

Plugging this C back in, then,

$$\frac{P^{-0.2}}{-0.2} = kt + \frac{P_0^{-0.2}}{-0.2}.$$

Finally, solve:

$$P^{-0.2} = -0.2kt + P_0^{-0.2}.$$

Or, since $0.2 = 1/5$,

$$P^{-1/5} = -\frac{kt}{5} + P_0^{-1/5} = \frac{1}{\sqrt[5]{P_0}} - \frac{kt}{5}.$$

Raise both sides to the -5 power:

$$\boxed{P = \left(\frac{1}{\sqrt[5]{P_0}} - \frac{kt}{5}\right)^{-5}}$$

Solution to the "supergrowth" IVP (SG)

Remark: under supergrowth P can, in theory, become infinite after a finite time!!

Why? Well, "$0^{-5} = \frac{1}{0^5} = \infty$," so in the above model, P becomes infinite when

$$\frac{1}{\sqrt[5]{P_0}} - \frac{kt}{5} = 0.$$

Solve for t:

$$\frac{kt}{5} = \frac{1}{\sqrt[5]{P_0}}$$

$$kt = \frac{5}{\sqrt[5]{P_0}}$$

$$\boxed{t = \frac{5}{k\sqrt[5]{P_0}}}$$

"Doomsday" for the IVP (SG)

Separation of variables, concluded.

Using the "separate, integrate, evaluate, solve" strategy, we can solve various DE's (IVP's) of the form

$$\frac{dy}{dx} = f(x)g(y), \qquad y(x_0) = y_0,$$

including applications to:

- Newton's Law of Cooling;
- Logistic Growth;
- Diffusion across a cell membrane (see HW 11);
- Supergrowth;
- Mixing;
- "Newton's Law of Warming."

} see Exam 4 "SVA" review sheet.