MATH 1310 Tutorial: Integrals and Accumulation functions SOLUTIONS

You have a large circular kiddie pool in your backyard. It has radius 10 feet, and is 8 feet deep. You are using a hose from your house to fill it to a depth of 6 feet.

Please supply the appropriate units with all of your answers below.

1. (Note: you should be able to do this first problem without any calculus.) Your hose supplies 10π cubic feet of water an hour (it's one of those fancy new π-hoses from late night TV).

   (a) What will the *depth* of the water be after 1 hour? after 2 hours? What are your units? Hint: recall that the volume of a cylinder is given by

   $$\text{volume} = (\text{depth}) \times (\text{area of the base}),$$

   and in this case the area of the base is $\pi r^2 = \pi \times 10^2 = 100\pi$. First compute the total volume of water after 1 hour or 2 hours, then use this hint to compute the depth.

   After 1 hour, the total *volume* of water in the pool will be 10π cubic feet/hour times 1 hour= 10π cubic feet. The area of the surface of the pool is $\pi(10)^2 = 100\pi$ square feet. So the depth after 1 hour is

   $$\frac{10\pi \text{ cubic feet}}{100\pi \text{ square feet}} = \frac{1}{10} \text{ feet} = 0.1 \text{ feet}.$$

   Similarly, the depth after 2 hours is

   $$\frac{20\pi \text{ cubic feet}}{100\pi \text{ square feet}} = \frac{1}{5} \text{ feet} = 0.2 \text{ feet}.$$

   (b) What will the depth of the water be after T hours? What are your units?

   Arguing as above, the volume of water after T hours will be $10\pi T$ cubic feet, so the depth after T hours will be

   $$\frac{10\pi T \text{ cubic feet}}{100\pi \text{ square feet}} = \frac{T}{10} \text{ feet} = 0.1\,T \text{ feet}.$$

(c) How long will it take to fill the pool to 6 feet? What are your units?
We solve $0.1\,T = 6$ for T, to find that $T = 6/0.1 = 60$ hours.

2. (Note: you probably **do** need calculus for this second question.) This is taking too long, so you buy the new Super Hose (only \$19.95 plus shipping and handling, with a free Ginsu Knife if you order before midnight tonight!), which supplies water at rate given by $w(t) = 10\pi t$ cubic feet of water per hour in hour t. (The longer the hose it on, the faster the water flows out of it!)

(a) What will the *volume* of water in the pool be after T hours? Your answer should be a function of T, call it $V(T)$. What are your units?

$$V(T) = \int_0^T w(t)\,dt = \int_0^T 10\pi t\,dt = 5\pi t^2\Big|_0^T = 5\pi\,T^2 \text{ cubic feet.}$$

(b) What will the *depth* of the water in the pool be after T hours? Your answer should be a function of T, call it $D(T)$. What are your units?

$D(T) = \dfrac{V(T)}{100\pi} = \dfrac{5}{100}\,T^2 = 0.05\,T^2$ feet.

(c) What will the depth of the water be after 1 hour? after 2 hours? What are your units?

$D(1) = 0.05$ feet; $D(2) = 0.05 \cdot 4 = 0.2$ feet.

(d) How long will it take to fill the pool to 6 feet? What are your units?

We solve $0.05\,T^2 = 6$ for T, to find that $T = \sqrt{6/0.05} = 10.9545$ hours.

(e) What is the rate of change of $D(T)$? What are your units?

$D'(T) = \dfrac{d}{dT}[0.05\,T^2] = 0.1T$ feet/hour.