Here we study relationships among numbers of vertices, edges, and faces in a polyhedron.

We begin with some definitions. First: a *simple closed surface* is an object, in three dimensions, that's hollow, has no holes, and does not cross itself anywhere. (That is: you can get from any point inside the surface to any other point inside the surface without crossing any part of the surface itself.) A *polyhedron* is a simple closed surface made up of polygonal regions joined together. (*Polyhedra* is the plural of polyhedron.)

The polygonal regions making up a polyhedron are called *faces* of the polyhedron. The terms *vertices* and *edges*, when applied to a polyhedron, refer simply to the vertices and edges of the polygonal regions making up that polyhedron.

1. First, you need to *build* the polyhedron from the "net" you've been supplied. When you're done, COUNT the vertices, edges, and faces of your polyhedron; then enter the name of your polyhedron, as well as the numbers you counted, in the indicated spaces of the first row below.

Name of Polyhedron	Number of vertices (V)	Number of edges (E)	Number of faces (F)
Octahedron	6	12	8
Truncated Square Pyramid	8	12	6
Step Pyramid	9	16	9
Pentagonal Pyramid	6	10	6

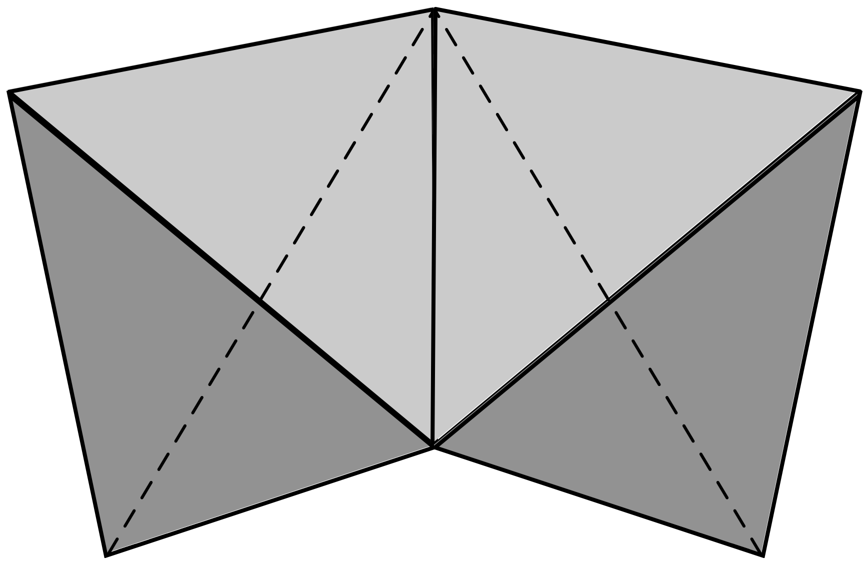
2. Using the information from the table you just completed, answer the following.

CONJECTURE ("Euler's Formula"): If P is any polyhedron, and V, E, and F represent the numbers of vertices, edges, and faces of P, respectively, then

$$\underline{\quad V - E + F \quad} = 2.$$

(The blank should be filled in with some simple expression involving V, E, and F, which you should be able to guess at by examining your table above.)

3. Use the sketch below of a "dual tetrahedron" (formed by gluing two tetrahedra together along an edge of each) to answer the given questions.



How many vertices does the dual tetrahedron have? ___6___ How many edges? ___11___
How many faces? ___8___

Does this contradict your conjecture above? Explain. Hint: carefully consider the definitions given at the beginning of this activity.

No, it does not contradict the conjecture, because the dual tetrahedron is not a polyhedron. Why? Because the definition of a polyhedron given on the first page says that *a polyhedron does not cross itself anywhere, meaning you can get from any point inside the surface to any other point inside the surface without crossing any part of the surface itself.* But this is not true for the dual tetrahedron, because to get from a point inside one of the tetrahedra to a point inside the other, you need to cross the surface of the object.