Your Spiral Design Tool (it's NOT a toy, this is SERIOUS!!) has three toothed gears and two toothed apertures. The gears have 36, 52, and 63 teeth respectively; the apertures have 96 and 105 teeth respectively.

1. For any given gear-and-aperture combination, let a denote the number of teeth in the aperture, and b the number of teeth in the gear. For every such combination, express a/b as a reduced fraction m/n. (That is, write a/b in reduced form.) Give your answers in the spaces below. (Use the space at the bottom of this page for scratch, if you wish.)

$a =$ 105 $b =$ 36 $\frac{a}{b} = \frac{35}{12}$ (in reduced form)

$a =$ 105 $b =$ 52 $\frac{a}{b} = \frac{105}{52}$ (in reduced form)

$a =$ 105 $b =$ 63 $\frac{a}{b} = \frac{5}{3}$ (in reduced form)

$a =$ 96 $b =$ 36 $\frac{a}{b} = \frac{8}{3}$ (in reduced form)

$a =$ 96 $b =$ 52 $\frac{a}{b} = \frac{24}{13}$ (in reduced form)

$a =$ 96 $b =$ 63 $\frac{a}{b} = \frac{32}{21}$ (in reduced form)

2. For any given gear-and-aperture combination, the numerator m and denominator n of the reduced fraction you found in problem 1 represent interesting features of the Spiral designs that the chosen combination produces. (These features depend ONLY on the choice of aperture and gear; they DON'T depend on which pinhole you choose to put your pen through.) What feature does m represent? What feature does n represent? Figure this out by actually drawing designs, using a couple of combinations (pick combinations that give relatively small numbers of petals, to make your life easier), and seeing what happens. (Hint: besides studying what your picture *looks like*, you should also keep track, as you're drawing, of how many times you go around the inside of the aperture.)

m represents: The number of petals of the design

n represents: The number of revolutions around the inside of the aperture required to complete the design

3. To mess with your head, your MATH 2300 instructor sneakily replaces your 52-toothed gear, when you're not looking, with one that looks almost exactly the same, but has a slightly different number of teeth. Now all of a sudden, the designs you draw with this gear, using the same 105-toothed and 96-toothed apertures as above, have 35 and 32 petals, respectively. How many teeth does your new gear have? Please explain.

It has 51. We need a number b, close to 52, such that $105/b$, in reduced form, has a 35 in the numerator, and $96/b$, in reduced form, has a 32 in the numerator. Some trial and error shows that $b = 51$ works: $105/51 = 35/17$; $96/51 = 32/17$.

4. In a top secret Spiro Lab, hidden somewhere in Roswell, New Mexico, Joe Biden and Khloe Kardashian have a huge Spiro set that features an aperture with 132,000 teeth and a gear with 13,750 teeth. How many petals will be on a design produced by this Spiro set? Please show your work, and express your answer as a whole number. Some hints: $132 = 2^2 \cdot 3 \cdot 11$; $1375 = 5^3 \cdot 11$.

It has 48 petals, since

$$\frac{132{,}000}{13{,}750} = \frac{2^2 \cdot 3 \cdot 11 \cdot 1{,}000}{5^3 \cdot 11 \cdot 10} = \frac{2^2 \cdot 3 \cdot 11 \cdot 100}{5^3 \cdot 11}$$
$$= \frac{2^2 \cdot 3 \cdot 11 \cdot 2^2 \cdot 5^2}{5^3 \cdot 11} = \frac{2^2 \cdot 3 \cdot 2^2}{5} = \frac{48}{5},$$

in reduced form.

5. Let's say you're drawing a spiral design with your 96-toothed aperture and your 36-toothed gear. Suppose you know that the *rate* at which your design is being traced out, in millimeters per *radian* (mm/rad), is given by a function $f(t)$. That is, for every Δt radians that you sweep out around the inside of the aperture, you draw roughly $f(t)\,\Delta t$ millimeters of curve (if Δt is small).

Write down an integral (involving $f(t)$) that will tell you the total length of your curve once you're done (that is, as soon as your pen returns to the starting point). You won't be able to evaluate this integral (since you don't know what $f(t)$ is). But you should at least specify your limits of integration.

We saw that $96/36 = 8/3$ in reduced form, so to complete the design, we must sweep around the aperture 3 times, for a total of $3 \cdot 2\pi = 6\pi$ radians. So the total length is

$$\int_0^{6\pi} f(t)\,dt \quad \text{mm.}$$