Your Spiral Design Tool (it's NOT a toy, this is SERIOUS!!) has three toothed gears and two toothed apertures. The gears have 35, 52, and 63 teeth respectively; the apertures have 96 and 105 teeth respectively.

**1.** For any given gear-and-aperture combination, let $a$ denote the number of teeth in the aperture, and $b$ the number of teeth in the gear. For every such combination, express $a/b$ as a reduced fraction $m/n$. (That is, write $a/b$ in reduced form.) Give your answers in the spaces below. (Use the space at the bottom of this page for scratch, if you wish.)

$a =$ 105 $b =$ 35 $\frac{a}{b} =$ $\frac{3}{1}$ (in reduced form)

$a =$ 105 $b =$ 52 $\frac{a}{b} =$ $\frac{105}{52}$ (in reduced form)

$a =$ 105 $b =$ 63 $\frac{a}{b} =$ $\frac{5}{3}$ (in reduced form)

$a =$ 96 $b =$ 35 $\frac{a}{b} =$ $\frac{96}{35}$ (in reduced form)

$a =$ 96 $b =$ 52 $\frac{a}{b} =$ $\frac{24}{13}$ (in reduced form)

$a =$ 96 $b =$ 63 $\frac{a}{b} =$ $\frac{32}{21}$ (in reduced form)

**2.** For any given gear-and-aperture combination, the numerator $m$ and denominator $n$ of the reduced fraction you found in problem 1 represent interesting features of the Spiral designs that the chosen combination produces. (These features depend ONLY on the choice of aperture and gear; they DON'T depend on which pinhole you choose to put your pen through.) What feature does $m$ represent? What feature does $n$ represent? Figure this out by actually drawing designs, using a couple of combinations (pick combinations that give relatively small numbers of petals, to make your life easier), and seeing what happens. (Hint: besides studying what your picture *looks like*, you should also keep track, as you're drawing, of how many times you go around the inside of the aperture.)

$m$ represents: The number of petals of the design

$n$ represents: The number of revolutions around the inside of the aperture required to complete the design

**3.** To mess with your head, your CSM instructor sneakily replaces your 63-toothed gear, when you're not looking, with one that looks almost exactly the same, but has a slightly different number of teeth. Now all of a sudden, the designs you draw with this gear, using the same 105-toothed and 96-toothed apertures as above, have 105 and 3 petals, respectively. How many teeth does your new gear have? Please explain.

It has 64. We need a number $b$, close to 63, such that $105/b$, in reduced form, has a 105 in the numerator, and $96/b$, in reduced form, has a 3 in the numerator. Some trial and error shows that $b = 64$ works: $105/64 = 105/64$; $96/64 = 3/2$.

**4.** In a top secret Spiro Lab, hidden somewhere in Roswell, New Mexico, Pharrell Williams and Khloe Kardashian have a huge Spiro set that features an aperture with 132,000 teeth and a gear with 13,750 teeth. How many petals will be on a design produced by this Spiro set? Please show your work, and express your answer as a whole number. Some hints: $132 = 2^2 \cdot 3 \cdot 11$; $1375 = 5^3 \cdot 11$.

It has 48 petals, since

$$\frac{132{,}000}{13{,}750} = \frac{2^2 \cdot 3 \cdot 11 \cdot 1{,}000}{5^3 \cdot 11 \cdot 10} = \frac{2^2 \cdot 3 \cdot 11 \cdot 100}{5^3 \cdot 11}$$
$$= \frac{2^2 \cdot 3 \cdot 11 \cdot 2^2 \cdot 5^2}{5^3 \cdot 11} = \frac{2^2 \cdot 3 \cdot 2^2}{5} = \frac{48}{5},$$

in reduced form.