Accumulation functions: recap (and more).

Suppose E is an accumulation function for p, meaning

ΔE over an interval $\approx p(t)\Delta t$

for Δt small (Δt = length of interval, t = any point in the interval).

(Think: E = energy, p = power.)

Then:
  A) $p(t) = E'(t)$.
  B) ΔE over any interval [a,b]
        = area of region below p(t) and above [a,b].

Combining (A) and (B) (and a small additional argument) tells us:

> If $p(t) = E'(t)$ then
> ΔE over [a,b]
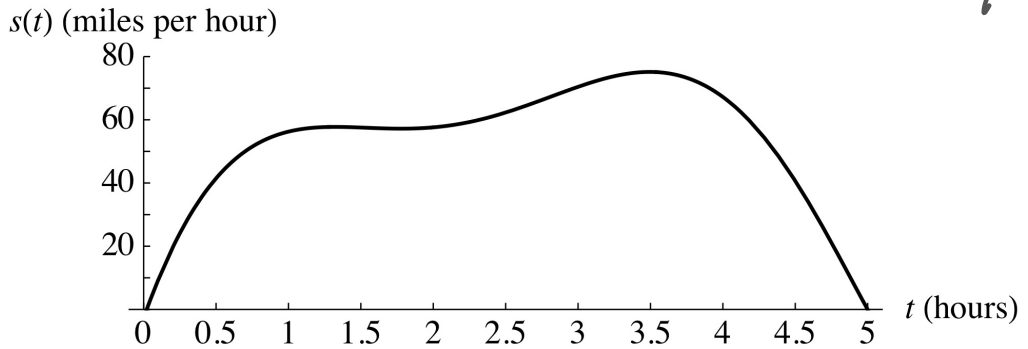>   = area of the region below p(t) and above [a,b]

The Fundamental Theorem of Calculus

Example:
    Here's a graph of a car's velocity against time:

s(t) (miles per hour)

Estimate the car's distance traveled over the 5 hours shown.

Solution.
  Using the rectangles shown below to approximate the area under the graph of v(t), we find that the distance traveled is about

$$0.5(40 + 55 + 55 + 54 + 61 + 67 + 72 + 70 + 44 + 0) = 259 \text{ mi.}$$

s(t) (miles per hour)