Week 10 - Tuesday, 10/27

Accumulation functions, continued.

Recall: if power $p(t)$ is constant on an interval of length Δt, then net energy ΔE over that interval is defined by

$$\Delta E = p(t)\,\Delta t, \qquad (*_=)$$

where t is any point on the interval.

This is just one case of:

Definition.

Suppose $E(t)$ and $p(t)$ are two functions such that $(*_=)$ holds whenever $p(t)$ is constant on an interval of length Δt (where t is any point on that interval).

Then E is called an accumulation function for p.

Other examples of accumulation functions:

E	p	t
Distance	velocity	time
Work	force	distance
Mass	density	length
Force	pressure	area
Total deaths	death rate	time

We explore two COOL FACTS about accumulation functions.

A) Accumulation and derivatives.

Suppose $p(t)$ is not constant on an interval: then $(*_=)$ need not hold. BUT: if the interval is narrow enough, then $p(t)$ won't change much on that interval, so we'd expect $(*_=)$ to almost hold; that is,

$$\Delta E \approx p(t)\Delta t \qquad (*_\approx)$$

for Δt small enough. Divide by Δt:

$$p(t) \approx \frac{\Delta E}{\Delta t} \quad \text{for } \Delta t \text{ small,}$$

where t is any point on the interval. The narrower the interval, the better the approximation, so:

$$p(t) = \lim_{\Delta t \to 0} \frac{\Delta E}{\Delta t}.$$

The right-hand side is just $E'(t)$!!
Conclusion:

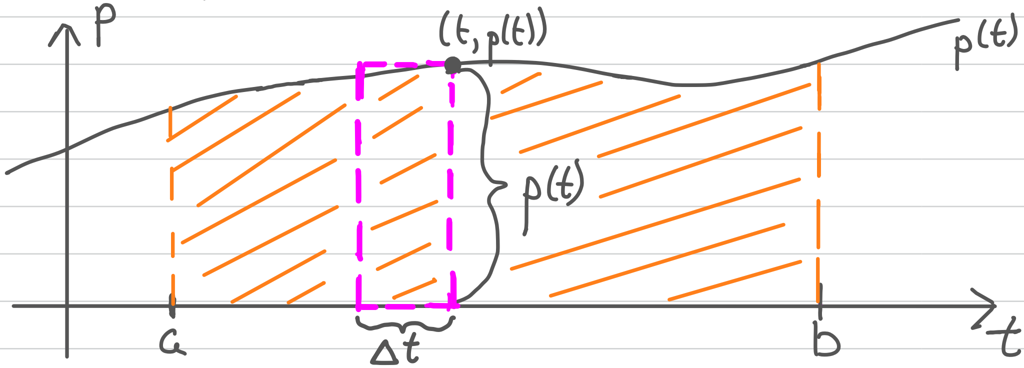
COOL FACT A.

If E is an accumulation function for p, then

$$p(t) = E'(t).$$

B) Accumulation and area.

Suppose E is an accumulation function for p. Say the graph of p looks like this:



We note that

ΔE over $[a,b]$,
$=$ sum of ΔE's over smaller intervals, like the one shown above
$\approx$ sum of products $p(t)\Delta t$, like the one shown above
$=$ sum of areas of rectangles like the one shown above
$\approx$ area (shaded in orange) under the graph of $p(t)$, over the interval $[a,b]$.

The shorter the intervals are, the better the approximations.

Conclusion:

$$\boxed{\begin{array}{l}\text{COOL FACT B.}\\ \quad\text{If } E \text{ is an accumulation function for } p,\\ \text{then}\\ \\ \quad \Delta E \text{ over } [a,b]\\ \quad = \text{area under the graph of } p, \text{ over the}\\ \qquad \text{interval } [a,b].\end{array}}$$