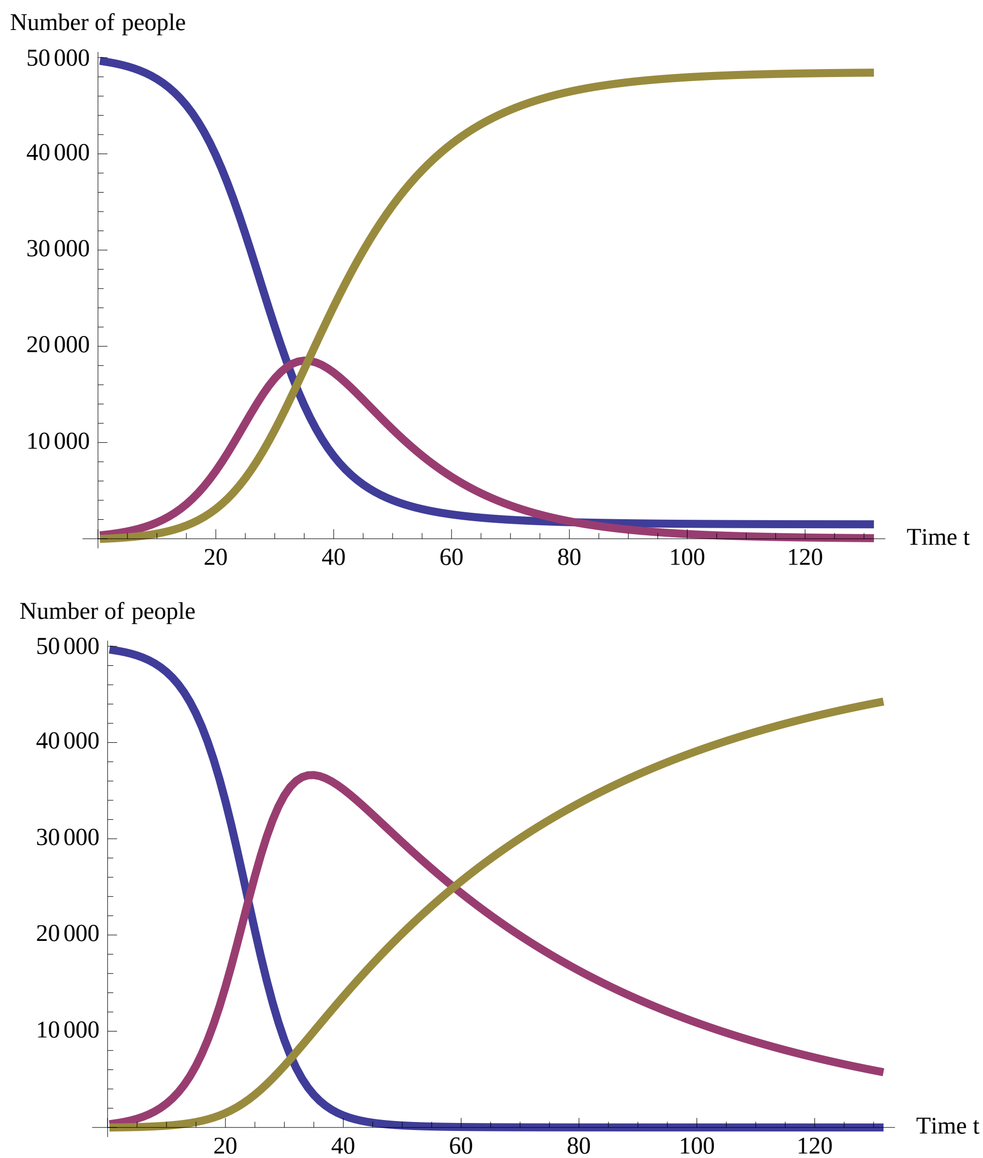
Goal: To explore some more ideas about modeling with rate equations and SIR.

1. Pictured below are two graphs depicting evolution of diseases that progress according to the usual SIR model. For both graphs, the initial values $S(0)$, $I(0)$ and $R(0)$, and the transmission coefficient a, are the same. But the two graphs correspond to different recovery coefficients b.



(a) On each of the graphs, label which curve is S, which is I, and which is R.

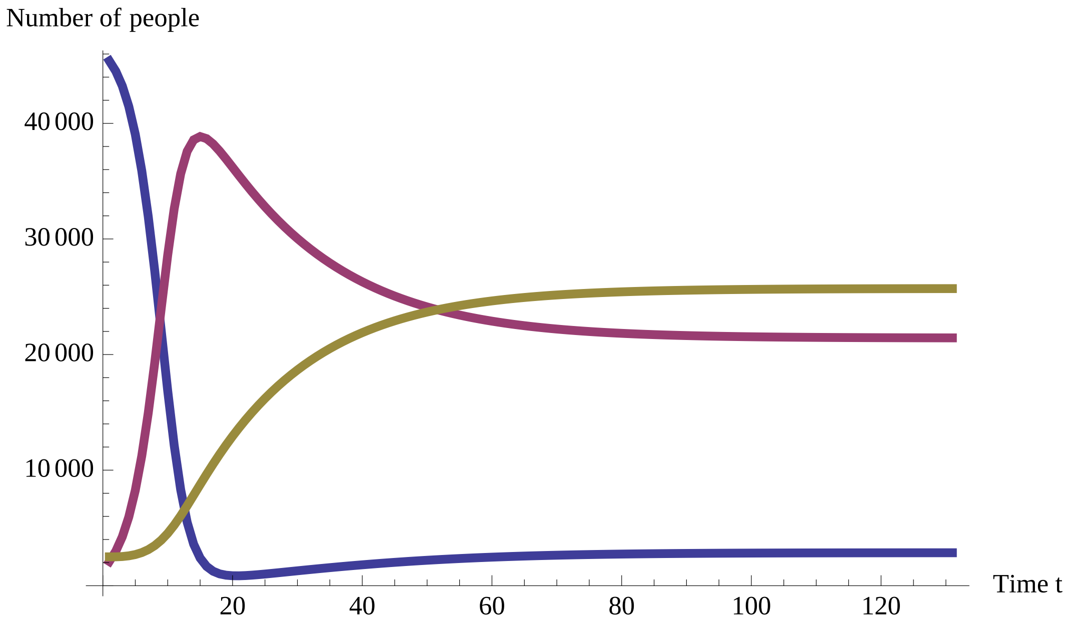
(b) Which of the above two graphs corresponds to the *larger* value of b? Please explain.

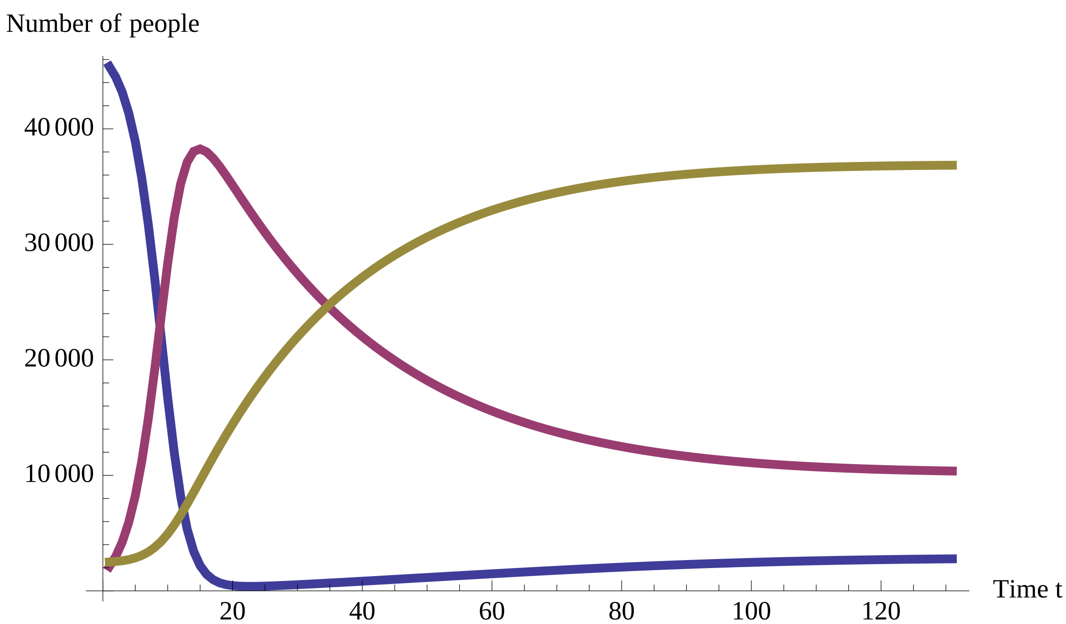
(c) Which of the above two epidemics has the larger basic reproduction number r_0? Please explain.

2. Consider an epidemic that progresses according to the usual SIR model, *except* that, now, recovered people become susceptible again (and can infect again) after m days.

(a) *Modify* the usual SIR equations to reflect this new feature (wherein recovered can become susceptible again). HINTS: (a) Your new equations will look a *lot* like the old ones, but with some *new terms* added on. These terms should account for the facts that, now, on average, $1/m$ of the recovered population gets *added to* susceptible population, and *subtracted from* the recovered population, on any given day. (b) Your new equations should involve unspecified parameters a, b, and c, where a and b are as above, and $c = 1/m$.

(b) In the two graphs on the next page, the transmission and recovery coefficients a and b are the same, but the number of days m that it takes to become susceptible again differs from one graph to the next. For which of the two graphs – the one on the top or the one on the bottom — does it take *longer* to become susceptible again? Please explain.


Number of people
40 000
30 000
20 000
10 000
20
40
60
80
100
120
Time t


Number of people
40 000
30 000
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10 000
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Time t