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The basic reproduction number r_0.

A) Definition and formula.

The basic reproduction number r_0 is the total number of new infections caused by each infected individual, at the outset of the epidemic.

Computation of r_0:

(a) Since $S' = -aSI$ there are, over the course of day 0, $aS(0)I(0)$ new infections. So:

(b) Per infected individual, there are

$$\frac{aS(0)I(0)}{I(0)} = aS(0)$$

new infections over the course of day 0. But one stays infected for k days, so:

(c) At the outset, there are $kaS(0)$ total new infections per infected individual.

Conclusion:

$$\boxed{r_0 = kaS(0)}$$

formula for the basic reproduction number

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(Here a = transmission coefficient, and $k = 1/b$, where b = recovery coefficient.)

<u>Example 1</u> In our epidemic with $S(0) = 500$, $a = 0.001$, $b = 0.2$, we have

$$r_0 = kaS(0) = \frac{1}{b} \cdot a \cdot S(0) = \frac{1}{0.2} \cdot 0.001 \cdot 500$$
$$= 2.5 \text{ individuals.}$$

(B) Important fact about r_0:

<u>Proposition.</u> If $r_0 > 1$, then I will initially grow.
If $r_0 < 1$, then I will initially shrink.

Proof: see text, p. 22.

(C) r_0 and herd immunity.

Recall from last time: herd immunity can be achieved by immunizing a fraction

$$f > 1 - \frac{b}{aS(0)} \qquad (*)$$

of the susceptible population. But $b = 1/k$, so $(*)$ can be written

$$f > 1 - \frac{1}{kaS(0)}$$ or, since $kaS(0) = r_0$,

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$$\boxed{f > 1 - \frac{1}{r_0}}$$ immunization proportion for herd immunity

<u>Example 2</u>. For our epidemic of Example 1, we attain herd immunity by immunizing better than

$$1 - \frac{1}{2.5} = 0.6 = 60\%$$

of the initial susceptible population. (Same result as in the notes for 9/1.)