Week 3 - Tuesday, 9/8

Linear functions.

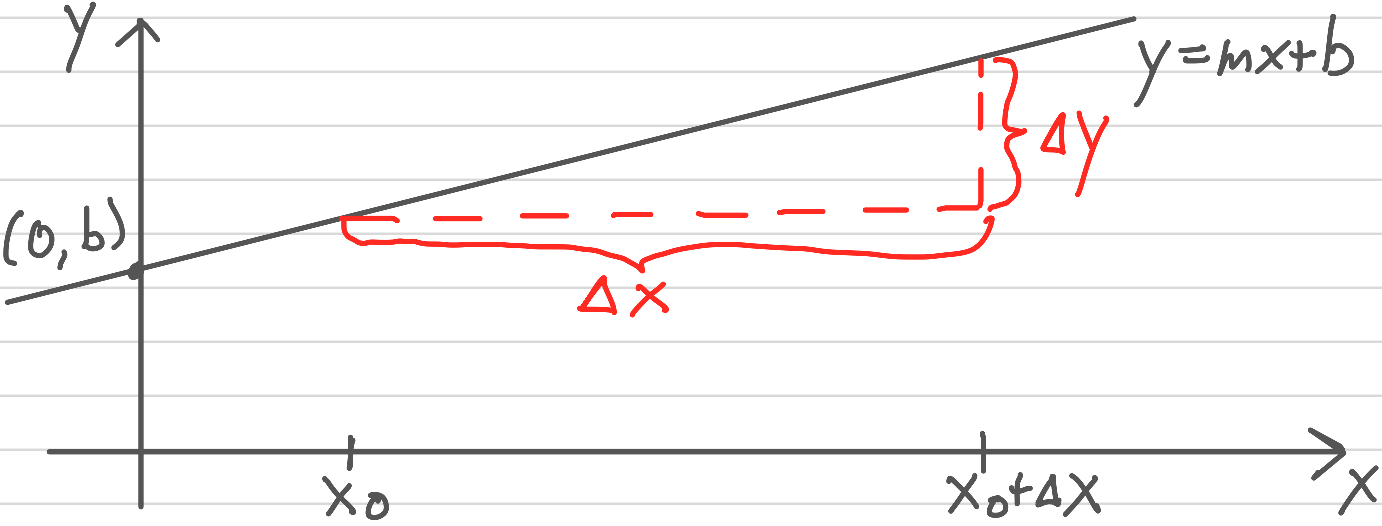
A) Basics.

A linear function (or line) is one of the form

$$y = mx + b.$$

the slope, or multiplier.

the y-intercept



COOL FACT about lines and rates of change:

Suppose $y = f(x) = mx + b$ is a linear function. If x changes by Δx, then by how much does y change?

Well, say x goes from x_0 to $x_0 + \Delta x$. Then the change in y is:

$$\begin{aligned}
\Delta y &= \text{new } y \text{ minus old } y \\
&= f(x_0 + \Delta x) - f(x_0) \\
&= m(x_0 + \Delta x) + b - (mx_0 + b) \\
&= \cancel{mx_0} + m\Delta x \cancel{+b} \cancel{-mx_0} \cancel{-b} \\
&= m\Delta x.
\end{aligned}$$

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<u>CONCLUSION.</u> For a linear function $y=mx+b$, Δy is always <u>proportional</u> to Δx:

$$\Delta y = m\Delta x, \quad \text{or} \quad m = \frac{\Delta y}{\Delta x}.^{*}$$

* Linear functions have a <u>constant</u> rate of change $\Delta y/\Delta x$, which equals the slope <u>m</u>. This is a <u>special</u> property of linear functions, as we'll see.

<u>Example.</u> Let C denote temperature in °C, and F temperature in °F.

(a) What's the multiplier m in the equation $\Delta F = m\Delta C$?
(b) What's the rate of change of F with respect to C?
(c) If C decreases by 2° C, by how much does F change?
(d) What's the rate of change of C with respect to F?

<u>Solution.</u>

(a) We have $F = \frac{9}{5}C + 32$. The slope m is $9/5$, so by the CONCLUSION above,

$\Delta F = \frac{9}{5}\Delta C$. So the multiplier is $9/5$.

(b) $m = 9/5$ (°F per °C)

(c) $\Delta F = \frac{9}{5}\Delta C = \frac{9}{5}\cdot(-2) = -18/5$.

F decreases by $18/5 = 3.6$ °F.

(d) We solve for C:

$$C = \frac{5}{9}(F-32) = \frac{5}{9}F - \frac{5\cdot 32}{9} = \frac{5}{9}F - \frac{160}{9}.$$

So the rate of change of C with respect to F is $5/9$ (°C per °F).

B) Equations for lines.

(1) As above: $\boxed{y = mx + b}$ slope-intercept form

Example. A line through (0,2), and such that y changes by −3 for each unit increase in x, has equation

$$y = -3x + 2.$$

(2) Say you're given the slope m of a line, and a point (x_0, y_0) on it. Then, for any other point (x, y) on the line,

$$m = \frac{\Delta y}{\Delta x} = \frac{y - y_0}{x - x_0}$$ or, solving for y,

$$\boxed{y = m(x - x_0) + y_0}$$ point-slope, or initial value, form.

Example. The line through (−2,1), with slope 4, has equation

$$\begin{aligned} y &= 4(x-(-2))+1 \\ &= 4x+8+1 = 4x+9. \end{aligned}$$

(3) "Two points determine a line."

If a line passes through (x_1, y_1) and (x_2, y_2), then by the point-slope form,

$$\boxed{y = m(x - x_1) + y_1 \quad \text{where } m = \frac{y_2 - y_1}{x_2 - x_1}.}$$ two-point, or interpolation, form

Example.

The line through (3,5) and (1,1) has slope

$$m = \frac{1-5}{1-3} = \frac{-4}{-2} = 2,$$

and equation

$$y = 2(x-3)+5$$
$$= 2x-6+5 = 2x-1.$$