MORE ON SIR.

I) Recall the SIR equations:

(SIR)
$$
\begin{aligned}
S' &= -aSI \\
I' &= aSI - bI \\
R' &= bI
\end{aligned}
$$

$a \, (>0)$: <u>transmission</u> coefficient. Units: $(person \cdot day)^{-1}$.
$b \, (>0)$: <u>recovery</u> coefficient. Units: day^{-1}.

Note: S is <u>decreasing</u> (since $S' < 0$).
I can increase and/or decrease.
R is <u>increasing</u> (since $R' > 0$).

Now, let's <u>use</u> (SIR) for:

II) Prediction.   HUGE IDEA:

if Q is any quantity varying with time, then, between any two instants (a "new" one and an "old" one), we have

(a) new Q = old $Q + \Delta Q$,

where ΔQ is the <u>change</u> in Q. Moreover, we have

(b) $\Delta Q = Q' \cdot \Delta t$ *

where Δt is elapsed time, and Q' is the <u>rate of change</u> of Q.
[(b) says: net change equals rate of change times elapsed time.]

III.) Let's now use (a), (b), and (SIR), as follows:

Example. Suppose we know that

$$S(0) = 500, \quad I(0) = 10, R(0) = 0,$$
$$a = 0.001, \quad b = 0.2.$$

(this denotes the value of S at time t=0)

(i) Predict $S(2), I(2), R(2)$.

Solution. Let's start with S. We have

$$
\begin{aligned}
S(2) &= S(0) + \Delta S && \text{(by (a) above)}\\
&= S(0) + S'(0) \cdot \Delta t && \text{(by (b) above)}\\
&= S(0) + (-a \cdot S(0) \cdot I(0)) \cdot \Delta t && \text{(by (SIR))}\\
&= 500 + (-0.001 \cdot 500 \cdot 10) \cdot 2 && \text{(plug in values)}\\
&= 500 - 10 = 490.
\end{aligned}
$$

We do R next ('cause it's easier than I):

$$
\begin{aligned}
R(2) &= R(0) + \Delta R\\
&= R(0) + R'(0) \cdot \Delta t\\
&= R(0) + b \cdot I(0) \cdot \Delta t\\
&= 0 + 0.2 \cdot 10 \cdot 2 = 4.
\end{aligned}
$$

To find $I(2)$, we recall that $S + I + R$ is constant, and initially equal to $500 + 10 + 0 = 510$, so

$$I(2) = 510 - S(2) - R(2) = 510 - 490 - 4 = 16.$$

Summary: according to the above model,

$$\boxed{S(2) = 490, \quad I(2) = 16, \quad R(2) = 4}$$

(ii) Use part (i) above to predict $S(4)$, $I(4)$, $R(4)$.

Solution.
$$S(4) = S(2) + \Delta S$$
$$= S(2) + S'(2)\Delta t$$
$$= S(2) + (-a \cdot S(2) \cdot I(2)) \cdot \Delta t$$
$$= 490 + (-0.001 \cdot 490 \cdot 16) \cdot 2 \quad \}\text{plug in the values}$$
$$= 474.32 \qquad\qquad\qquad\qquad\qquad \text{computed in Part (i) above}$$

Similarly one finds $R(4)$ and $I(4)$; the net result is

$$\boxed{S(4) = 474.32, \quad I(4) = 25.28, \quad R(4) = 10.4.}$$

* NOTE: the "=" in the equation
$$\Delta Q = Q' \Delta t$$
(see II(b) above) should really be "$\approx$" (approximately equals).
Why? Because Q itself changes with time. More on this idea soon.