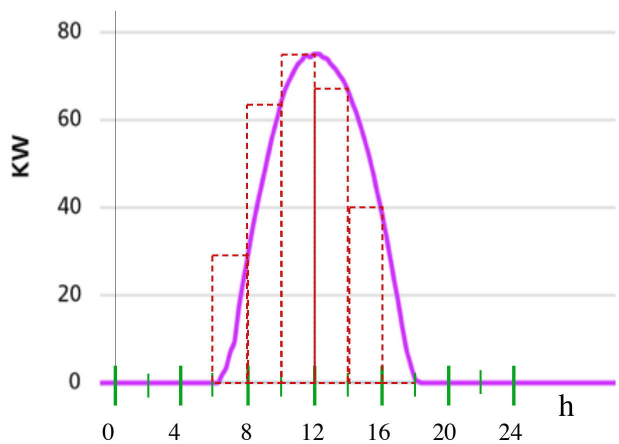
Individual Homework #6: Due Saturday, November 14

Section 4.1:

2. (a) $1{,}000 \times 2 + 1{,}500 \times 3 + 500 \times 9 = 11{,}000$ watt-hours.

4. (a)



(b) $E(0) \approx E(2) \approx E(4) \approx E(6) \approx 0$, $E(8) \approx 60$, $E(10) \approx 60 + 128 = 188$, $E(12) \approx 188 + 152 = 340$, $E(14) \approx 340 + 136 = 476$, $E(16) \approx 476 + 80 = 556$, $E(18) \approx E(20) \approx E(22) \approx E(24) \approx 456$ (kilowatt-hours).

(d) To get a better approximation, use narrower rectangles for the approximation. For example, sample every half-hour instead of every two hours.

(e) $E'(T) = p(T)$, so the graph of E' looks like the original power function graph. (That is, it looks like the bell-shaped curve that you started with.)

Section 4.2, Part 1:

2. (a) The combined weight is $W(x) = 2{,}000 + 40(30 - x)$ pounds.

(b) At the bottom of the first ten-foot interval, $x = 0$, so the weight is $W(0) = 2{,}000 + 40(30 - 0) = 3{,}200$ pounds. An estimate for the work done over this interval is then

$$\text{force} \times \text{distance} = 3{,}200 \text{ pounds} \times 10 \text{ feet} = 32{,}000 \text{ foot-pounds}.$$

At the bottom of the second ten-foot interval, $x = 10$, so the weight is $W(10) = 2{,}000 + 40(30 - 10) = 2{,}800$ pounds. An estimate for the work done over this interval is then

$$2{,}800 \text{ pounds} \times 10 \text{ feet} = 28{,}000 \text{ foot-pounds}.$$

Finally, At the bottom of the third ten-foot interval, $x = 20$, so the weight is $W(20) = 2{,}000 + 40(30 - 20) = 2{,}400$ pounds. An estimate for the work done over this interval is then

$$2{,}400 \text{ pounds} \times 10 \text{ feet} = 24{,}000 \text{ foot-pounds.}$$

An estimate for the total work done is therefore

$$32{,}000 + 28{,}000 + 24{,}000 = 84{,}000 \text{ foot-pounds.}$$

(c) This is similar to part (b), except that now, the "sampling points" are $x = 10$, $x = 20$, and $x = 30$. So an estimate for the total work done is

$$\begin{aligned}
&W(10) \cdot 10 + W(20) \cdot 10 + W(30) \cdot 10 \\
&= (2{,}000 + 40(30 - 10)) \times 10 + (2{,}000 + 40(30 - 20)) \times 10 + (2{,}000 + 40(30 - 30)) \times 10 \\
&= 28{,}000 + 24{,}000 + 20{,}000 = 72{,}000 \text{ foot-pounds.}
\end{aligned}$$

(d) $(84{,}000 + 72{,}000)/2 = 78{,}000$ foot-pounds.

Section 4.3, Part 1:

1. (a)
$$\begin{aligned}
A &\approx f(3) \cdot 1 + f(4) \cdot 1 + f(5) \cdot 1 + f(6) \cdot 1 = \sqrt{1 + 3^3} + \sqrt{1 + 4^3} + \sqrt{1 + 5^3} + \sqrt{1 + 6^3} \\
&= 5.2915 + 8.0623 + 11.2250 + 14.7309 = 39.3097.
\end{aligned}$$

   (b) It's an underestimate. The function is increasing on the interval, and as noted earlier, an increasing function has underestimating left endpoint Riemann sums.

   (c) $A \approx 52.5654$. This is an overestimate.

3.
$$\begin{aligned}
A &\approx f(1.25) \cdot \frac{1}{2} + f(1.75) \cdot \frac{1}{2} + f(2.25) \cdot \frac{1}{2} + \cdots + f(5.75) \cdot \frac{1}{2} \\
&= \sqrt{1.25 - 1} + \sqrt{1.75 - 1} + \sqrt{2.25 - 1} + \cdots + \sqrt{5.75 - 1} \\
&= 7.45356.
\end{aligned}$$

Section 4.3, Part 2:

4. The estimates are 43.18768, 45.55509, 45.7935, 45.81736. To the nearest hundredth, the approximations seem to be converging to 45.82.

5. 5.84720, 6.19124, 6.22608, 6.22957. 6.23.

8. (a) 45.8106, 45.81992, 45.82001, 45.82001. 45.82. (b) The midpoint Riemann sums seem to be much more efficient.

Section 4.4, Part 2:

7. (a) 12.8714; stabilizes after $n = 150$ rectangles.

Section 4.5, Part 1:

1. (a) 1. (b) 474.6666. (d) 0. (f) 1.7918. (h) 1.4427. (i) 0.6667. (j) 0.

2. $\displaystyle\int_{-2}^{4} x\,dx = \frac{x^2}{2}\bigg|_{-2}^{4} = \frac{4^2-(-2)^2}{2} = 6$. Also:

$$\int_{-\pi}^{\pi} \sin(x)\,dx = -\cos(x)\big|_{-\pi}^{\pi} = -\cos(\pi) - (-\cos(-\pi)) = -(-1) + (-1) = 0.$$

Section 4.5, Part 2:

5.

$$\begin{aligned}\int_0^{24} 1000\left(1+\cos\left(\frac{\pi}{12}t\right)\right)dt &= 1000\left(t + \frac{12}{\pi}\sin\left(\frac{\pi}{12}t\right)\right)\bigg|_0^{24} \\ &= 1000(24 + \frac{12}{\pi}\sin(2\pi)) - 1000(0 + \frac{12}{\pi}\sin(0)) \\ &= 1000(24+0) - 1000(0+\sin(0)) = 24{,}000.\end{aligned}$$

6. The distance traveled is

$$\begin{aligned}&\int_0^5 (127t - 90t^2 + 17.35t^3 + 5t^4 - 2.079t^5 + 0.18t^6)\,dt \\ &= \left(\frac{127t^2}{2} - \frac{90t^3}{3} + \frac{17.35t^4}{4} + \frac{5t^5}{5} - \frac{2.079t^6}{6} + \frac{0.18t^7}{7}\right)\bigg|_0^7 \\ &= \frac{127\cdot 5^2}{2} - \frac{90\cdot 5^3}{3} + \frac{17.35\cdot 5^4}{4} + \frac{5\cdot 5^5}{5} - \frac{2.079\cdot 5^6}{6} + \frac{0.18\cdot 5^7}{7} = 268.304 \text{ miles.}\end{aligned}$$