CLS Homework #2 Notes and Common Errors

Homework Due: 1 February 2019
Problems Graded: 1.5.7, 2.1.1, and 2.2.1

- General Notes -

1. Please make sure that your work flow is clear and easy to follow. There were a couple of people that had really messy work that was a bit difficult to follow and grade.

- Problem 1.5.7 -

1. Most people did pretty well on this. There was, however, a little bit of confusion regarding the sign of ΔW. Most people got that $W(3) = 162$ and $W(7) = 92$, which does give us that W **decreases** by 68 units. Therefore, we should get $\Delta W = -68$.
2. Be careful with your algebra. There were a couple of small mistakes in subtraction that ended up costing people points.

- Problem 2.1.1 -

1. I think that there was a little bit of confusion on how, exactly, we wanted you all to answer parts (b) and (d) of this problem, so I have provided a fully-worked solution below. Please take a look at it, and let me know if you have further questions.

Q: Let $f(x) = 2x^2 - 3$.

(a) Find the average rate of change $\Delta y/\Delta x$ of $f(x)$ with respect to x, from $x = 2$ to $x = 2 + \Delta x$, for each of the following three values of Δx: $\Delta x = 0.1,\ \Delta x = 0.01,\ \Delta x = 0.001$.

(b) Based on part (a) above, what might you guess $f'(2)$ is equal to?

(c) Use algebra to show that the average rate of change of $f(x)$ with respect to x, from $x = 2$ to $x = 2 + \Delta x$, is $8 + 2\Delta x$.

(d) Find the instantaneous rate of change of $f(x)$ at $x = 2$.

(e) Find the equation of the line tangent to the graph of $f(x)$ at $x = 2$.

Solution:

(a) We know that the average rate of change $\Delta y/\Delta x$ from x to $x + \Delta x$ is given by the equation

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$

So for $\Delta x = 0.1$ we have that

$$\begin{aligned}
\frac{\Delta y}{\Delta x} &= \frac{f(x+\Delta x) - f(x)}{\Delta x} \\
&= \frac{f(2+0.1) - f(2)}{0.1} \\
&= \frac{f(2.1) - f(2)}{0.1} \\
&= \frac{[2(2.1)^2 - 3] - [2(2^2) - 3]}{0.1} \\
&= \frac{[8.82 - 3] - [8 - 3]}{0.1} \\
&= \frac{5.82 - 5}{0.1} \\
&= \frac{0.82}{0.1} \\
&= \boxed{8.2}
\end{aligned}$$

Similar calculations for $\Delta x = 0.01$ and $\Delta x = 0.001$ should give us estimates of 8.02 and 8.002 respectively.

(b) Looking at the results of part (a), we can see that as Δx gets closer and closer to 0, the average rate of change is getting closer and closer to 8. Since the value of $f'(2)$ is the limit as Δx goes to 0 of $\Delta y/\Delta x$, we can guess that as Δx goes to 0, $\Delta y/\Delta x$ will go to 8, giving us that $f'(2) \approx 8$ (I use the squiggly line here to denote that this is a guess).

(c) We know that the average rate of change of $f(x)$ at $x = a$ is given by

$$\frac{\Delta y}{\Delta x} = \frac{f(a+\Delta x) - f(a)}{\Delta x}.$$

Evaluating this, we get that

$$\begin{aligned}
\frac{\Delta y}{\Delta x} &= \frac{f(2+\Delta x) - f(2)}{\Delta x} \\
&= \frac{[2(2+\Delta x)^2 - 3] - [2(2)^2 - 3]}{\Delta x} \\
&= \frac{[2(4 + 4\Delta x + (\Delta x)^2) - 3] - [8 - 3]}{\Delta x} \\
&= \frac{[8 + 8\Delta x + 2(\Delta x)^2 - 3] - 5}{\Delta x} \\
&= \frac{[8\Delta x + 2(\Delta x)^2 + 5] - 5}{\Delta x} \\
&= \frac{8\Delta x + 2(\Delta x)^2}{\Delta x} \\
&= \frac{\Delta x \cdot (8 + 2\Delta x)}{\Delta x} \\
&= \boxed{8 + 2\Delta x}
\end{aligned}$$

(d) We know that the instantaneous rate of change of $f(x)$ at $x = 2$ is the limit as Δx goes to 0 of the average rate of change $\Delta y/\Delta x$ of $f(x)$ at $x = 2$. Well, in part (c) we computed that the average rate of change of $f(x)$ at $x = 2$ was $8 + 2\Delta x$, so we have that

$$f'(2) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(2+\Delta x) - f(2)}{\Delta x} = \lim_{\Delta x \to 0} 8 + 2\Delta x = \boxed{8}$$

(e) To find the equation of the line tangent to the graph of $f(x)$ at $x = 2$ we only need to evaluate $f(2)$ and $f'(2)$. Well $f(2) = 5$, which means that the tangent line passes through the point $(2, 5)$ and $f'(2) = 8$ which means that the slope of the tangent line is $m = 8$. Using the point-slope form of a line, we get that the equation of the tangent line is

$$\boxed{y = 8(x-2) + 5}$$

or equivalently

$$\boxed{y = 8x - 11}$$

- Problem 2.2.1 -

1. The graph was locally linear or differentiable. Just linear implies that the graph resembles a straight line no matter how far you zoom out.
2. The slope of the graph should have been 180, but I accepted anything reasonably close to that because eyeballing isn't an exact science. This should have also been your estimate for $f'(2)$.