Solutions to Selected Exercises, Individual Homework #3

Assignment:
Section 2.1 (pages 65–67): Exercises 1, 4, 7.
Section 2.2 (pages 72–73): Exercises 1, 3, 4.
Section 2.3, Part 2: Differentiation using the definition of the derivative (pages 85–87): Exercise 6.
Section 2.3, Part 3: Differentiation using rules and formulas (pages 87–89): Exercises 12, 14ace, 15.

Section 2.1

1(a). $\Delta y/\Delta x = 8.2,\ 8.02,\ 8.002$. **(b).** 8.
(c).

$$\frac{\Delta y}{\Delta x} = \frac{f(2+\Delta x) - f(2)}{\Delta x} = \frac{2(2+\Delta x)^2 - 3 - (2\times 2^2 - 3)}{\Delta x}$$
$$= \frac{8 + 8\Delta x + 2(\Delta x)^2 - 3 - 5}{\Delta x} = \frac{8\Delta x + 2(\Delta x)^2}{\Delta x} = \frac{\Delta x(8 + 2\Delta x)}{\Delta x} = 8 + 2\Delta x.$$

(d). $f'(2) = \lim_{\Delta x\to 0}(8 + 2\Delta x) = 8$. (d) $y = f(2) + f'(2)(x-2) = 5 + 8(x-2) = 8x - 11$.

7(a). $\dfrac{\Delta y}{\Delta x} = \dfrac{f(64+\Delta x) - f(64)}{\Delta x} = \dfrac{\sqrt{64+\Delta x} - \sqrt{64}}{\Delta x} = \dfrac{\sqrt{64+\Delta x} - 8}{\Delta x}$.
(b).

$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{64+\Delta x} - 8}{\Delta x} = \frac{\sqrt{64+\Delta x} - 8}{\Delta x} \times \frac{\sqrt{64+\Delta x} + 8}{\sqrt{64+\Delta x} + 8}$$
$$= \frac{(\sqrt{64+\Delta x})^2 + 8\sqrt{64+\Delta x} - 8\sqrt{64+\Delta x} - 8^2}{\Delta x(\sqrt{64+\Delta x} + 8)} = \frac{64 + \Delta x - 64}{\Delta x(\sqrt{64+\Delta x} + 8)}$$
$$= \frac{\Delta x}{\Delta x(\sqrt{64+\Delta x} + 8)} = \frac{1}{\sqrt{64+\Delta x} + 8}.$$

(c).

$$f'(64) = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{1}{\sqrt{64+\Delta x} + 8} = \frac{1}{\sqrt{64} + 8} = \frac{1}{8+8} = \frac{1}{16}.$$

Section 2.2

1(a). $f(x)$ looks more and more like a line. **(b).** Locally linear (or differentiable). **(d).** It looks roughly like the graph passes through the points $(1.99, 54.2)$ $(2.01, 57.8)$. This gives us a slope of $(57.8 - 54.2)/(2.01 - 1.99) = 180.0$. So we estimate that $f'(2) \approx 180.0$.

4. The graph does not flatten out as we zoom in. This is because $g(x)$ is *not* locally linear at $x = 2$. It *is* locally linear at every other point, though. For example, if $x > 2$, then the absolute values go away, so $g(x) = x^2 - 10(x-2) = x^2 - 10x + 20$, which is just a polynomial. If $x < 2$, then $|x-2| = -(x-2) = -x+2$, so $g(x) = x^2 - 10(-x+2) = x^2 + 10x - 20$, which is another polynomial.

Section 2.3

6(a).

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}} \\ &= \lim_{\Delta x \to 0} \frac{(\sqrt{x+\Delta x} - \sqrt{x})(\sqrt{x+\Delta x} + \sqrt{x})}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} \\ &= \lim_{\Delta x \to 0} \frac{(\sqrt{x+\Delta x})^2 - \sqrt{x}\sqrt{x+\Delta x} + \sqrt{x}\sqrt{x+\Delta x} - (\sqrt{x})^2}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} \\ &= \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} \\ &= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}. \end{aligned}$$

(b.) $\sqrt{x} = x^{1/2}$ and $1/\sqrt{x} = x^{-1/2}$, so this exercise tells us that $\frac{d}{dx}[x^{1/2}] = \frac{1}{2}x^{-1/2}$, which is the case $p = 1/2$ of the power formula.

12. (a). $f'(x) = 21x^6 - 1.2x^3 + 3\pi x^2$

(b). $\frac{d}{dx}\left[\sqrt{3}\,\sqrt{x} + \frac{7}{x^5}\right] = \frac{\sqrt{3}}{2\sqrt{x}} - \frac{35}{x^6}$

(c). $h'(w) = \frac{2}{3}w^7 - \cos(w) - \frac{2}{3w^3}$

(d). $\frac{d}{du}\left[\frac{4\cos(u)}{5} - \frac{3\tan(u)}{8} + \sqrt[3]{u}\right] = -\frac{4}{5}\sin(u) - \frac{3}{8}\sec^2(u) + \frac{1}{3u^{2/3}}$

(e). $V'(s) = -\frac{1}{4s^{3/4}}$

(f). $F'(z) = \sqrt{7}\ln(2)2^z + \ln(1/2)(1/2)^z$

(g). $P'(t) = -at + v_0$

14(a). $f(x) = x^{12}$. (c) $f(x) = \sin(x) - \cos(x)$.

15. 3/4. 19/20. 199/200.